CHAPTER 6

Exponents and Polynomials

Section 6A

Exponents

- 6-1 Integer Exponents
- 6-2 Rational Exponents

Section 6B

Polynomials

- 6-3 Polynomials
  - Algebra Lab Model Polynomial Addition and Subtraction
- 6-4 Adding and Subtracting Polynomials
  - Algebra Lab Model Polynomial Multiplication
- 6-5 Multiplying Polynomials
- Connecting Algebra to Geometry Volume and Surface Area
- Extension Closure
- 6-6 Special Products of Binomials

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 6

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
6-1 Lesson	6-2 Lesson	Multi-Step Test Prep Ready to Go On?	6-3 Lesson	Algebra Lab 6-4 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
6-4 Lesson	Algebra Lab 6-5 Lesson	6-5 Lesson	Extension	Connecting Algebra to Geometry
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	
6-6 Lesson	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test	

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 6

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
6-1 Lesson 6-2 Lesson	Multi-Step Test Prep Ready to Go On? 6-3 Lesson	Algebra Lab 6-4 Lesson	Algebra Lab 6-5 Lesson	Extension Connecting Algebra to Geometry
DAY 6	**DAY 7**			
6-6 Lesson Multi-Step Test Prep Ready to Go On?	Chapter Review Chapter Test			

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 6

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

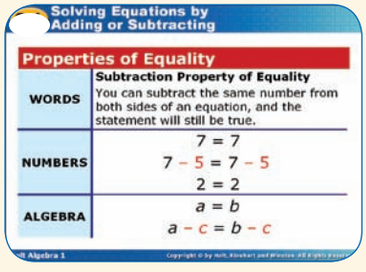
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

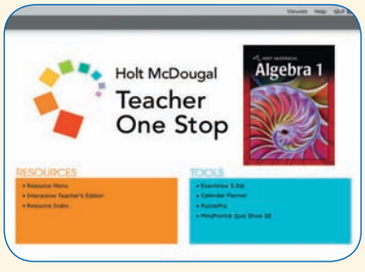
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 6

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Critical Thinking ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

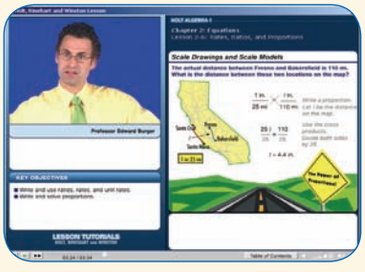
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

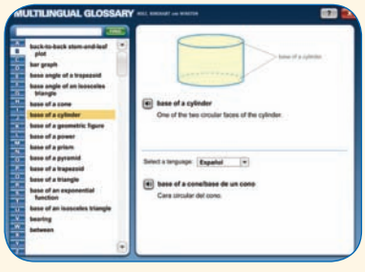
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



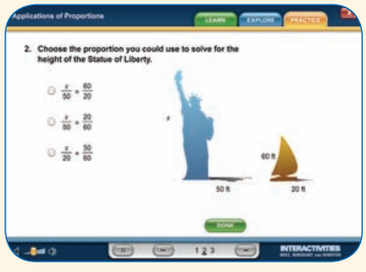
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 6

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!® Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!® Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView™ Assessment Suite

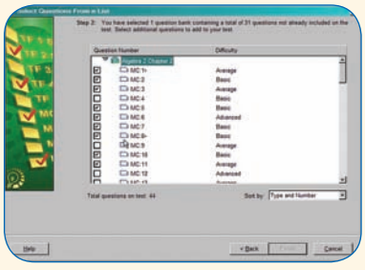
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView™ Assessment Suite
IDEA Works!® Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 6

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

B Chapter Test

Select the best answer.

1. Which of the following is equivalent to 2^{-3}?
   A $(-2)(-2)(-2)$ C $\frac{1}{(2)(2)(2)}$
   B $-\frac{1}{(2)(2)(2)}$ D $(2)(2)(2)$
2. Evaluate $(3 + x)^{-2}$ for $x = -1$.
   F -4 H $\frac{10}{9}$
   G $\frac{1}{4}$ J 10
3. Simplify $4b^{-3}$.
   A $-64b^3$ C $\frac{1}{4b^3}$
   B $\frac{1}{64b^3}$ D $\frac{4}{b^3}$
4. Simplify 2^4.
   F 12 H 32
   G 16 J 64
5. Simplify $\left(\frac{1}{216}\right)^0$.
   A 0 C 6
   B 1 D 36
6. Simplify 4^{-4}.
   F -256 H $\frac{1}{256}$
   G $-\frac{1}{256}$ J 256
7. Simplify $256^{\frac{1}{2}}$.
   A 4 C 64
   B 16 D 128
8. Simplify $125^{\frac{2}{3}}$.
   F -5 H 25
   G 5 J 125
9. Simplify $\left(\frac{1}{3}\right)^{-4}$.
   A -3 C 9
   B $-\frac{1}{81}$ D 81
10. Simplify $\left(\frac{1}{64}\right)^{\frac{1}{3}}$.
    F $\frac{1}{8}$ H 4
    G $\frac{1}{4}$ J 8
11. Simplify $(-8)^{\frac{4}{3}}$.
    A -16
    B -2
    C 2
    D 16
12. Simplify $\left(\frac{4}{9}\right)^{\frac{1}{2}}$.
    F 81 H $\frac{3}{2}$
    G $\frac{4}{3}$ J $\frac{2}{9}$
13. Simplify $\left(\frac{5}{b^4}\right)^{-2}$.
    A $\frac{1}{25b^4}$ C $\frac{b^8}{25}$
    B $\frac{5}{b^2}$ D $-5b^2$
14. Simplify $\left(\frac{1}{6}\right)^3$.
    F 216 H $-\frac{1}{216}$
    G 36 J $-\frac{1}{36}$

B Chapter Test *(continued)*

15. Simplify $256^{\frac{1}{4}}$.
    A 4 C 6
    B 5 D 64
16. Simplify $64^{\frac{2}{3}}$.
    F 2 H 16
    G 8 J 512
17. Simplify $(x^{\frac{1}{2}})^4\sqrt{x^3}$. All variables represent nonnegative numbers.
    A x^2 C $x^{2(x)}$
    B x^3 D x^8
18. When written in standard form, which polynomial has a leading coefficient of 5?
    F $-7 + 6y + 5y^2$
    G $x + 5$
    H $x^2 - 5x^3 + 2x$
    J $5y + 3y^2 - 4$
19. Classify the polynomial $3x^5 + 3$ according to its degree and number of terms.
    A cubic binomial
    B cubic trinomial
    C quintic binomial
    D quintic trinomial
20. Brett has 100 feet of fence with which to make a rectangular cage for his dog. The area of the cage in square feet is given by the polynomial $-w^2 + 50w$, where w is the width of the cage in feet. What is the area of the cage if the width is 8 feet?
    F 114 ft^2 H 384 ft^2
    G 336 ft^2 J 464 ft^2
21. Add $(2x^2 - 5x - 7) + (7x^2 + 3)$.
    A $2x^2 + 2x - 4$
    B $9x^2 - 5x - 4$
    C $9x^2 - 2x - 7$
    D $9x^4 - 5x - 4$
22. Subtract $(7a^2 - 3a) - (5a^2 - 5a)$.
    F $2a^2 - 8a$ H 4
    G $2a^2 + 2a$ J $12a^2 - 8a$
23. Multiply $(2x + 1)(x - 1)$.
    A $3x$
    B $2x^2 - 1$
    C $2x^2 - x - 1$
    D $2x^2 - 3x - 1$
24. A rectangle has width w and its length is 2 units shorter than 3 times the width, or $3w - 2$. Write a polynomial for the area of the rectangle.
    F $3w^2 - 2$ H $4w - 2$
    G $3w^2 - 2w$ J $8w - 4$
25. Multiply $(x - 5)(2x + 4)$.
    A $-6x$ C $2x^2 - 20$
    B $2x^2 - 26$ D $2x^2 - 6x - 20$
26. Multiply $(b + 3)(b^2 - 5b - 7)$.
    F $b^3 - 5b^2 - 21$
    G $b^3 - 2b^2 - 22b - 21$
    H $3b^3 - 15b^2 - 21b$
    J $4b^3 - 20b^2 - 28b$
27. Multiply $(2x + 7)^2$.
    A $2x^2 + 7$ C $4x^2 + 14x + 49$
    B $4x^2 + 49$ D $4x^2 + 28x + 49$
28. Which product results in $x^2 - 100$?
    F $(x - 10)^2$
    G $(x + 10)^2$
    H $x(x - 100)$
    J $(x + 10)(x - 10)$

A Chapter Test

C Chapter Test

FREE RESPONSE

B Chapter Test

1. Simplify 4^{-3}.
2. Evaluate $(5 - x)^{-2}$ for $x = -1$.
3. Simplify $5b^{-8}$.
4. Simplify 13^2.
5. Simplify 74^1.
6. Simplify 1^{π}.
7. Simplify 5^0.
8. Simplify $32^{\frac{1}{5}}$.
9. Simplify $\left(\frac{1}{81}\right)^{\frac{1}{4}}$.
10. Simplify $\left(\frac{1}{10}\right)^y$.
11. Simplify $625^{\frac{3}{4}}$.
12. Simplify $\left(\frac{1}{16}\right)^{\frac{1}{2}}$.
13. $\left(\frac{2}{b^5}\right)^{-3}$

B Chapter Test *(continued)*

14. Simplify 4^{-4}.

Simplify.

15. $32^{\frac{1}{5}}$
16. $27^{\frac{4}{3}}$
17. Simplify $(x^{\frac{1}{3}})^2\sqrt[3]{y^3}$. All variables represent nonnegative numbers.
18. When the polynomial $5x - 2x^3 + 8x^2 - 7$ is written in standard form, what is the leading coefficient?
19. Classify the polynomial $5x^2 + 9x + 1$ according to its degree and number of terms.
20. Genie has 100 feet of fence with which to make a rectangular cage for her rabbit. If she uses the wall of her house as one side, the area of the cage in square feet is given by the polynomial $-2w^2 + 100w$, where w is the width of the cage in feet. What is the area of the cage if the width is 15 feet?
21. Add $(5x^2 - 2x + 9) + (2x^2 - 4)$.
22. Subtract $(10a^2 - 6a) - (7a^2 - 8a)$.
23. Multiply $(3x + 1)(x + 1)$.
24. A rectangle has width w and its length is 4 units longer than 2 times the width, or $2w + 4$. Write a polynomial for the area of the rectangle.

Multiply.

25. $(x + 6)(3x - 8)$
26. $(b - 4)(b^2 + 3b - 2)$
27. $(3x - 4)^2$
28. $(2x + 4)(2x - 4)$

PERFORMANCE ASSESSMENT

Chapter Test

Radicals and Fractional Exponents.

Purpose

This performance task assesses the student's ability to change between fractional exponents and radicals.

Time

30–40 minutes

Grouping

Individuals

Preparation Hints

Review radicals and fractional exponents. Review how to change between the two forms.

Overview

Students first rewrite fractional exponents as radicals. Then they write radicals as fractional exponents. Students will have to change negative exponents, exponents with numerators greater than 1, cube and fourth roots.

Many of the problems can be simplified, but students must demonstrate that they can change between radicals and fractional exponents and understand the relationship between the two. Simplified answers are, therefore, incorrect.

Introduce the Task

As a class, go over the provided examples. Mention that radicands can not be negative; therefore, when converting negative fractional exponents into radicals, place the radical in the denominator and remember the students should not simplify.

Performance Indicators

____ Rewrites fractional exponents as radicals correctly.
____ Rewrites radicals as fractional exponents correctly.
____ Converts between the two forms without simplifying.

Scoring Rubric

Level 4: Student solves all problems correctly and gives a solid comparison between radicals and fractional exponents.
Level 3: Student solves most problems correctly but gives a faulty comparison between radicals and fractional exponents.
Level 2: Student solves a few problems and gives no comparison.
Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

Radicals and Fractional Exponents.

Fractional exponents can be written as radicals, and radicals can be written as fractional exponents. Use the examples below to help you fill in the blanks.

$16^{\frac{1}{2}} = \sqrt{16}$

$16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}}$

$16^{\frac{3}{2}} = \sqrt{16^3}$

Write these fractional exponents as radicals.

1. $2^{\frac{1}{2}}$
2. $16^{\frac{3}{4}}$
3. $\left(\frac{1}{8}\right)^{\frac{1}{3}}$
4. $25^{\frac{1}{2}}$
5. $16^{-\frac{1}{2}}$
6. $\left(\frac{1}{36}\right)^{-\frac{1}{2}}$

Write these radicals as fractional exponents.

7. $\sqrt{6}$
8. $\frac{1}{\sqrt{25}}$
9. $\sqrt{27^2}$
10. $\sqrt{4^3}$
11. $\sqrt{81}$
12. $\sqrt{4^7}$

CHAPTER 6

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

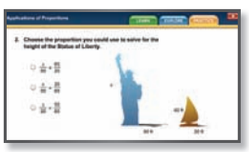
SECTION 6A
Exponents

MULTI-STEP TEST PREP Students write, solve, and graph equations to model real-world speed-of-light situations.

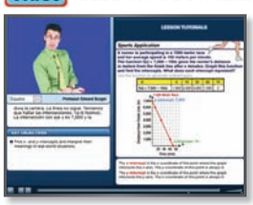
SECTION 6B
Polynomials

MULTI-STEP TEST PREP Students multiply polynomials to model a real-world area situation.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 6

Exponents and Polynomials

6A Exponents

6-1	Integer Exponents	N.RN.1
6-2	Rational Exponents	N.RN.1

6B Polynomials

6-3	Polynomials	A.SSE.1a
Lab	Model Polynomial Addition and Subtraction	A.APR.1
6-4	Adding and Subtracting Polynomials	A.APR.1
Lab	Model Polynomial Multiplication	A.APR.1
6-5	Multiplying Polynomials	A.APR.1
Ext	Closure	A.APR.1
6-6	Special Products of Binomials	A.APR.1

COMMON CORE

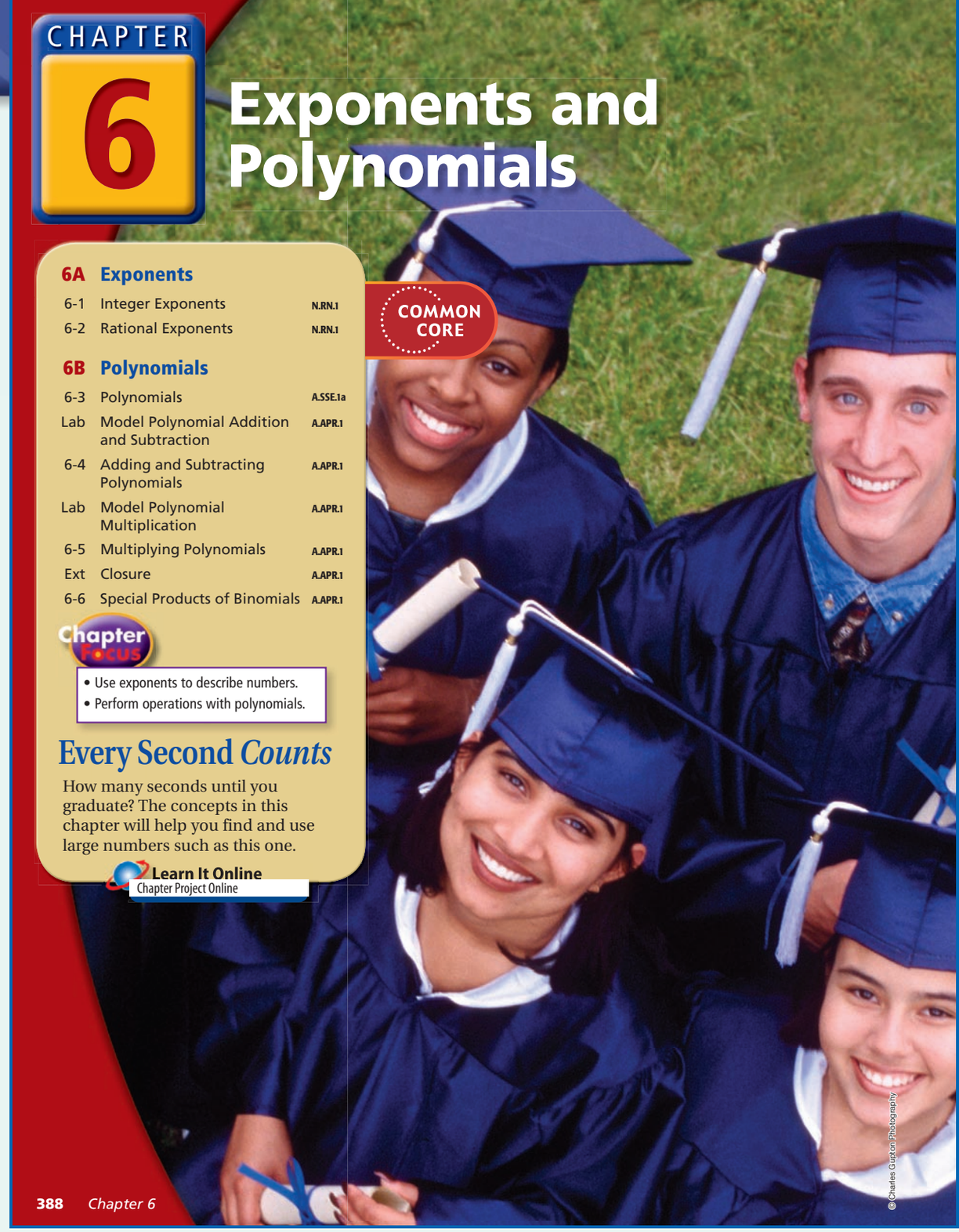
Chapter Focus

- Use exponents to describe numbers.
- Perform operations with polynomials.

Every Second *Counts*

How many seconds until you graduate? The concepts in this chapter will help you find and use large numbers such as this one.

Learn It Online
Chapter Project Online



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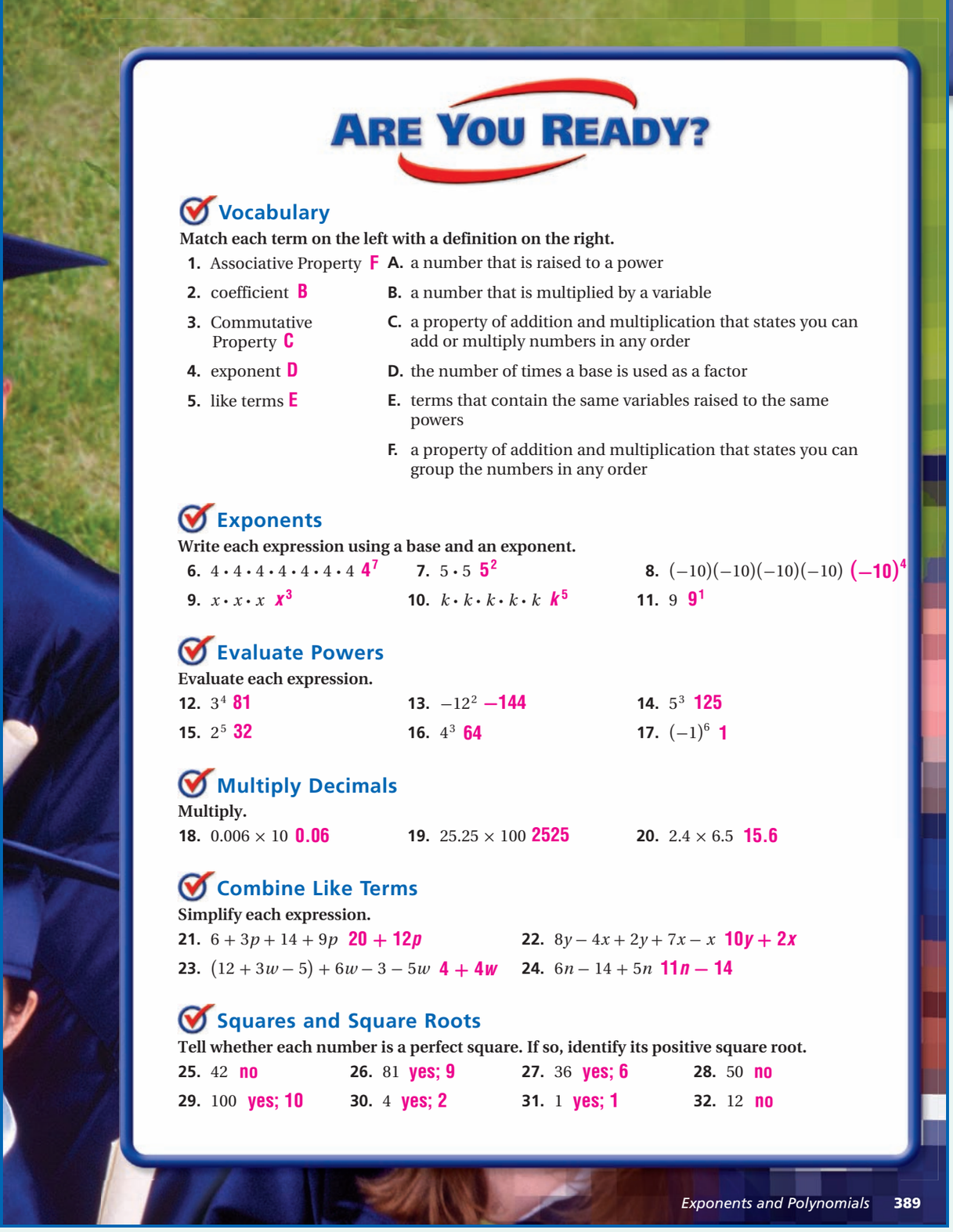
Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. Associative Property **F**
2. coefficient **B**
3. Commutative Property **C**
4. exponent **D**
5. like terms **E**

A. a number that is raised to a power

B. a number that is multiplied by a variable

C. a property of addition and multiplication that states you can add or multiply numbers in any order

D. the number of times a base is used as a factor

E. terms that contain the same variables raised to the same powers

F. a property of addition and multiplication that states you can group the numbers in any order

Exponents

Write each expression using a base and an exponent.

6. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **4^7**
7. $5 \cdot 5$ **5^2**
8. $(-10)(-10)(-10)(-10)$ **$(-10)^4$**
9. $x \cdot x \cdot x$ **x^3**
10. $k \cdot k \cdot k \cdot k \cdot k$ **k^5**
11. 9 **9^1**

Evaluate Powers

Evaluate each expression.

12. 3^4 **81**
13. -12^2 **−144**
14. 5^3 **125**
15. 2^5 **32**
16. 4^3 **64**
17. $(-1)^6$ **1**

Multiply Decimals

Multiply.

18. 0.006×10 **0.06**
19. 25.25×100 **2525**
20. 2.4×6.5 **15.6**

Combine Like Terms

Simplify each expression.

21. $6 + 3p + 14 + 9p$ **$20 + 12p$**
22. $8y - 4x + 2y + 7x - x$ **$10y + 2x$**
23. $(12 + 3w - 5) + 6w - 3 - 5w$ **$4 + 4w$**
24. $6n - 14 + 5n$ **$11n - 14$**

Squares and Square Roots

Tell whether each number is a perfect square. If so, identify its positive square root.

25. 42 **no**
26. 81 **yes; 9**
27. 36 **yes; 6**
28. 50 **no**
29. 100 **yes; 10**
30. 4 **yes; 2**
31. 1 **yes; 1**
32. 12 **no**

CHAPTER **6**

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

***Are You Ready? Intervention and Enrichment* Worksheets**

***Are You Ready?* Online**

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention,* Chapter 6 Worksheets	*Are You Ready? Intervention,* Chapter 6 Online	
Exponents	Skill 7	Activity 7	Diagnose and Prescribe Online
Evaluate Powers	Skill 8	Activity 8	
Multiply Decimals	Skill 45	Activity 45	
Combine Like Terms	Skill 57	Activity 57	
Squares and Square Roots	Skill 6	Activity 6	

***Are You Ready? Enrichment,* Chapter 6**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Answers to *Vocabulary Connections*

1. the coefficient of the first (lead) term, which is the number attached to the variable of that term
2. an expression with exactly 2 terms
3. Possible answers: tricycle, triangle, tripod; they all have three of something; an expression with exactly three terms.

CHAPTER 6 Study Guide: Preview

Where You've Been

Previously, you

- wrote and evaluated exponential expressions.
- simplified algebraic expressions by combining like terms.

In This Chapter

You will study

- how to evaluate and simplify expressions containing exponents.
- how to add, subtract, and multiply polynomials by using properties of exponents and combining like terms.

Where You're Going

You can use the skills in this chapter

- to model area, perimeter, and volume in geometry.
- to express very small or very large quantities in science classes such as Chemistry, Physics, and Biology.
- in the real world to model business profits and population growth or decline.

Key Vocabulary/Vocabulario

binomial	binomio
closure	cerradura
degree of a monomial	grado de un monomio
degree of a polynomial	grado de un polinomio
element	elemento
leading coefficient	coeficiente principal
monomial	monomio
perfect-square trinomial	trinomio cuadrado perfecto
polynomial	polinomio
set	conjunto
standard form of a polynomial	forma estándar de un polinomio
subset	subconjunto
trinomial	trinomio

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. A **polynomial** written in standard form may have more than one algebraic term. What do you think the **leading coefficient** of a polynomial is?
2. A simple definition of **monomial** is "an expression with exactly one term." If the prefix *mono-* means "one" and the prefix *bi-* means "two," define the word **binomial**.
3. What words do you know that begin with the prefix *tri-*? What do they all have in common? Define the word **trinomial** based on the prefix *tri-* and the information given in Problem 2.

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 396, 401–402, 404, 409–411, 417–419, 427–429, 435, 438–439, 440
2. **Reason abstractly and quantitatively.** Pages 403, 410, 419, 428–429
3. **Construct viable arguments and critique the reasoning of others.** Pages 394, 396, 400, 402–403, 408, 410–411, 416, 418, 426, 428–429, 437, 438–439
4. **Model with mathematics.** Pages 416, 426, 435
5. **Use appropriate tools strategically.** Pages 412, 420, 423
6. **Attend to precision.** Pages 401, 402, 410, 429
7. **Look for and make use of structure.** Pages 397, 401–403, 411, 418, 428–429, 438–439, 434
8. **Look for and express regularity in repeated reasoning.** Pages 392, 400, 431–432

390 Chapter 6

Reading Strategy: Read and Understand the Problem

Follow this strategy when solving word problems.

- Read the problem through once.
- Identify exactly what the problem asks you to do.
- Read the problem again, slowly and carefully, to break it into parts.
- Highlight or underline the key information.
- Make a plan to solve the problem.

29. Multi-Step Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

Step 1	**Identify exactly what the problem asks you to do.**	• Show and describe the number of hours Linda can work at each job and earn at least $90 per week, without working more than 20 hours per week. • List two possible solutions of the system.
Step 2	**Break the problem into parts. Highlight or underline the key information.**	• Linda has two jobs. She makes **$15 per hour** at one job and **$10 per hour** at the other job. • She wants to earn **at least $90 per week.** • She does **not** want to work **more than 20 hours per week.**
Step 3	**Make a plan to solve the problem.**	• Write a system of inequalities. • Solve the system. • Identify two possible solutions of the system.

Try This

For the problem below,

a. identify exactly what the problem asks you to do.

b. break the problem into parts. Highlight or underline the key information.

c. make a plan to solve the problem.

1. The difference between the length and the width of a rectangle is 14 units. The area is 120 square units. Write and solve a system of equations to determine the length and the width of the rectangle. (*Hint:* The formula for the area of a rectangle is $A = \ell w$.)

Standards for Mathematical Content High School

		LESSON
THE REAL NUMBER SYSTEM Extend the properties of exponents to rational exponents		
CC.9-12.N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	6-1, 6-2
CC.9-12.N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	6-2
SEEING STRUCTURE IN EXPRESSIONS Interpret the structure of expressions		
CC.9-12.A.SSE.1a	Interpret parts of an expression, such as terms, factors, and coefficients.	6-3
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS Perform arithmetic operations on polynomials		
CC.9-12.A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Lab 6-4, 6-4, Lab 6-5, 6-5, 6-5 Ext, 6-6

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE **Online Edition**

ENGLISH LANGUAGE LEARNERS

Reading Strategy: Read and Understand the Problem

Discuss Key information can include more than just numbers. Words and phrases such as *not, no more than, whole number solutions, inches*, and *two different ways* can be crucial to correctly solving a problem.

Extend As you present word problems in this chapter to the class, ask students what information should be highlighted and why.

Answers to *Try This*

1a. Find the length and width of the rectangle.

b. The difference between length and width is 14 units. The area is 120 square units.

c. Write and solve a system of equations.

Reading Connection

Powers of Ten
by Philip and Phylis Morrison

This is a wondrous yet practical guide to powers of 10 and exponential growth. The book begins with one billion light years. In 40 steps, each step depicting $\frac{1}{10}$ the scale of the previous step, it moves to subatomic particles.

Activity Ask students to make their own books using 1 ft (10^0 ft) as step 6, 10^1 ft as step 7, 10^2 ft as step 8, and so on, up to step 11. Then 10^{-1} ft will be step 5, 10^{-2} ft will be step 4, etc. For each step, have students name (and perhaps locate a photo of) an object near that length.

SECTION
6A Exponents

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 6-1 Integer Exponents • Evaluate expressions containing zero and integer exponents. • Simplify expressions containing zero and integer exponents. ☑ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** graphing calculator
Lesson 6-2 Rational Exponents • Evaluate and simplify expressions containing rational exponents. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests		**Optional** graphing calculator, number cubes

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 79, 81, 83, 87, 89, 91, 93, 95, 97

Lesson 2: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 47, 51, 55, 59, 61, 63

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 57, 61, 63, 67, 69, 51, 73, 75a, 75b, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 39, 41, 43, 45, 47, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71

Lesson 5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85a, 85b, 87, 89, 91, 93, 95, 97

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

EXPONENTS

Lesson 6-1

Up to this point, students have worked primarily with linear equations and linear inequalities. In this chapter, students move toward more complex ideas as they begin to study polynomials. Before beginning this study, students must first have an understanding of exponents.

One common difficulty students have with exponents is the use of zero. For example, students are often puzzled by the fact that any nonzero number raised to the zero power is 1. It makes sense to think of 2^4 as a product where 2 is a factor 4 times ($2^4 = 2 \cdot 2 \cdot 2 \cdot 2$), but when it comes to evaluating 2^0, how does one write a product with 2 as a factor zero times? Students should understand that 2^0 is *defined* to be 1 in order to make it consistent with the rules of exponent arithmetic. For example, in order for the Quotient of Powers Property to work in as many situations as possible, it must be true that $\frac{2^4}{2^4} = 2^{4-4} = 2^0$, but $\frac{2^4}{2^4} = 1$. Thus, $2^0 = 1$.

This idea of defining certain powers in order to create a system that is as widely consistent as possible also explains why the expression 0^0 is undefined. First, it is clear that $0^1 = 0$, $0^2 = 0$, and $0^{13} = 0$. In fact, for any value of n greater than zero, $0^n = 0$. For this reason, it might make sense to define 0^0 as zero. On the other hand, as shown above, any nonzero number raised to the zero power is 1, so it might also make sense to define 0^0 as 1. Because there is no single real number that works consistently as a definition of 0^0, this expression is considered *indeterminate* and is left undefined.

6-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Evaluate expressions containing zero and integer exponents.

Simplify expressions containing zero and integer exponents.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression for the given values of the variables.

1. x^3y^2 for $x = -1$ and $y = 10$
-100

2. $\frac{3x^2}{y^2}$ for $x = 4$ and $y = -7$
$\frac{48}{49}$

Write each number as a power of the given base.

3. 64; base 4 4^3

4. -27; base -3 $(-3)^3$

Also available online

Math Fact !!!

The Babylonian number system had no symbol for zero. It was represented by a blank space.

CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Learn It Online
State Resources Online

COMMON CORE

6-1 Integer Exponents

CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Objectives
Evaluate expressions containing zero and integer exponents.

Simplify expressions containing zero and integer exponents.

Who uses this?
Manufacturers can use negative exponents to express very small measurements.

In 1930, the Model A Ford was one of the first cars to boast precise craftsmanship in mass production. The car's pistons had a diameter of $3\frac{7}{8}$ inches; this measurement could vary by at most 10^{-3} inch.

You have seen positive exponents. Recall that to simplify 3^2, use 3 as a factor 2 times: $3^2 = 3 \cdot 3 = 9$.

But what does it mean for an exponent to be negative or 0? You can use a table and look for a pattern to figure it out.

Remember!
Base x^4 Exponent

Power	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}
Value	3125	625	125	25	5			

$\div 5$ $\div 5$ $\div 5$ $\div 5$

When the exponent decreases by one, the value of the power is divided by 5. Continue the pattern of dividing by 5:

$5^0 = \frac{5}{5} = 1$ $\quad 5^{-1} = \frac{1}{5} = \frac{1}{5^1}$ $\quad 5^{-2} = \frac{1}{5} \div 5 = \frac{1}{25} = \frac{1}{5^2}$

Know it! Note

Integer Exponents

WORDS	NUMBERS	ALGEBRA
Zero exponent—Any nonzero number raised to the zero power is 1.	$3^0 = 1$ $\quad 123^0 = 1$ $(-16)^0 = 1$ $\quad \left(\frac{3}{7}\right)^0 = 1$	If $x \neq 0$, then $x^0 = 1$.
Negative exponent—A nonzero number raised to a negative exponent is equal to 1 divided by that number raised to the opposite (positive) exponent.	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$	If $x \neq 0$ and n is an integer, then $x^{-n} = \frac{1}{x^n}$.

Reading Math
2^{-4} is read "2 to the negative fourth power."

Notice the phrase "nonzero number" in the table above. This is because 0^0 and 0 raised to a negative power are both undefined. For example, if you use the pattern given above the table with a base of 0 instead of 5, you would get $0^0 = \frac{0}{0}$. Also, 0^{-6} would be $\frac{1}{0^6} = \frac{1}{0}$. Since division by 0 is undefined, neither value exists.

Advertising Archive

1 Introduce

EXPLORATION

Integer Exponents

A botanist has taken over a study of a plant whose height doubles every day. On the fourth day after the experiment began, the plant was 16 inches tall.

1. The botanist can find the height of the plant on previous days by repeatedly dividing by 2. Use this fact to complete the middle column of the table.
2. Use your knowledge of exponents to fill in the right column for days 1 and 2. Then look for patterns to complete the right column.
3. What does day 0 represent? What was the height of the plant on day 0? How can you write the height as a power?
4. What does day -1 represent? What was the height of the plant on day -1? How can you write the height as a power?

Day	Height of Plant (in.)	Height Written as a Power
4	16	2^4
3	8	2^3
2	4	2^2
1	2	2^1
0	1	2^0
-1	$\frac{1}{2}$	2^{-1}
-2	$\frac{1}{4}$	2^{-2}
-3	$\frac{1}{8}$	2^{-3}

THINK AND DISCUSS

5. **Describe** the pattern in the right column of the table.
6. **Show** how you could find the height of the plant on day -4 and then write the height as a power.
7. the day he took over the experiment, 4 in. = 2^2

Motivate

Show students the following examples and ask them to suggest a rule about the use of negative exponents.

a. $2^{-1} = \frac{1}{2}$ **b.** $2^{-3} = \frac{1}{8}$ **c.** $5^{-2} = \frac{1}{25}$

d. $3^{-2} = \frac{1}{9}$

Possible answer: The negative exponent means that you must use the reciprocal of the base and change the exponent to a positive number.

Explorations and answers are provided in your online resources.

EXAMPLE 1 Manufacturing Application

The diameter for the Model A Ford piston could vary by at most 10^{-3} inch. Simplify this expression.

$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000}$

10^{-3} inch is equal to $\frac{1}{1000}$ inch, or 0.001 inch.

CHECK IT OUT! 1. A sand fly may have a wingspan up to 5^{-3} m. Simplify this expression. $\frac{1}{125}$ m

EXAMPLE 2 Zero and Negative Exponents

Simplify.

A 2^{-3}

$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

B 5^0

$5^0 = 1$ *Any nonzero number raised to the zero power is 1.*

C $(-3)^{-4}$

$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)(-3)(-3)(-3)} = \frac{1}{81}$

D -3^{-4}

$-3^{-4} = -\frac{1}{3^4} = -\frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = -\frac{1}{81}$

Caution!

In $(-3)^{-4}$, the base is negative because the negative sign is inside the parentheses.

In -3^{-4} the base (3) is positive.

CHECK IT OUT! Simplify.

2a. 10^{-4} $\frac{1}{10{,}000}$ 2b. $(-2)^{-4}$ $\frac{1}{16}$ 2c. $(-2)^{-5}$ $-\frac{1}{32}$ 2d. -2^{-5} $-\frac{1}{32}$

EXAMPLE 3 Evaluating Expressions with Zero and Negative Exponents

Evaluate each expression for the given value(s) of the variable(s).

A x^{-1} for $x = 2$

2^{-1} *Substitute 2 for x.*

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ *Use the definition $x^{-n} = \frac{1}{x^n}$.*

B a^0b^{-3} for $a = 8$ and $b = -2$

$8^0 \cdot (-2)^{-3}$ *Substitute 8 for a and –2 for b.*

$1 \cdot \frac{1}{(-2)^3}$ *Simplify expressions with exponents.*

$1 \cdot \frac{1}{(-2)(-2)(-2)}$ *Write the power in the denominator as a product.*

$1 \cdot \frac{1}{-8}$ *Simplify the power in the denominator.*

$-\frac{1}{8}$ *Simplify.*

CHECK IT OUT! Evaluate each expression for the given value(s) of the variable(s).

3a. p^{-3} for $p = 4$ **3b.** $8a^{-2}b^0$ for $a = -2$ and $b = 6$

3a. $\frac{1}{64}$

3b. 2

COMMON ERROR ALERT

Students will sometimes multiply the base by the negative exponent. Have these students reread the definition of a negative exponent. Point out that 10^{-3} means a number less than one, not a number less than zero.

Power Presentations with PowerPoint®

Additional Examples

Example 1

One cup is 2^{-4} gallons. Simplify this expression. $\frac{1}{16}$ gal

Example 2

Simplify.

A. 4^{-3} $\frac{1}{64}$ **B.** 7^0 1

C. $(-5)^{-4}$ $\frac{1}{625}$ **D.** -5^{-4} $-\frac{1}{625}$

Example 3

Evaluate each expression for the given value(s) of the variable(s).

A. x^{-2} for $x = 4$ $\frac{1}{16}$

B. $-2a^0b^{-4}$ for $a = 5$ and $b = -3$ $-\frac{2}{81}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLES 1–2

- What is the difference between **Examples 2C** and **2D**?
- When will a term with a negative exponent have a negative value?

EXAMPLE 3

- There are no fractions in the problem. Why are there fractions in the answers?
- Why is the value of a variable irrelevant if that variable is raised to the zero power?

Teaching Tip **Inclusion** In **Example 3,** some students may prefer to rewrite the expression with a positive exponent before substituting for the variable.

2 Teach

Guided Instruction

Explain negative and zero exponents by demonstrating the pattern in the decreasing values of the powers. Use the definition to simplify and evaluate expressions with negative and zero exponents. Then include examples with negative exponents in the denominator. Remind students that factors with negative exponents should be simplified.

Teaching Tip **Technology** Students can use the ^ or Y^x keys on their calculators to check their work in **Examples 1–3.**

Reaching All Learners *Through Graphic Organizers*

Have students make a chart similar to the one below and let them refer to it during class work and homework.

0^x $(x \leq 0)$	Undefined
x^0 $(x \neq 0)$	1
x^{-2} $(x \neq 0)$	$\frac{1}{x^2}$
$\frac{1}{x^{-2}}$ $(x \neq 0)$	x^2

Power Presentations with PowerPoint®

Additional Examples

Example 4

Simplify.

A. $7w^{-4}$ $\frac{7}{w^4}$ **B.** $\frac{-5}{k^{-2}}$ $-5k^2$

C. $\frac{a^0b^{-2}}{c^{-3}d^6}$ $\frac{c^3}{b^2d^6}$

Also available online

What if you have an expression with a negative exponent in a denominator, such as $\frac{1}{x^{-8}}$?

$x^{-n} = \frac{1}{x^n}$, or $\frac{1}{x^n} = x^{-n}$ *Definition of negative exponent*

$\frac{1}{x^{-8}} = x^{-(-8)}$ *Substitute −8 for n.*

$= x^8$ *Simplify the exponent on the right side.*

So if a base with a negative exponent is in a denominator, it is equivalent to the same base with the opposite (positive) exponent in the numerator.

An expression that contains negative or zero exponents is not considered to be simplified. Expressions should be rewritten with only positive exponents.

EXAMPLE 4 Simplifying Expressions with Zero and Negative Exponents

Simplify.

A $3y^{-2}$

$3y^{-2} = 3 \cdot y^{-2}$

$= 3 \cdot \frac{1}{y^2}$

$= \frac{3}{y^2}$

B $\frac{-4}{k^{-4}}$

$\frac{-4}{k^{-4}} = -4 \cdot \frac{1}{k^{-4}}$

$= -4 \cdot k^4$

$= -4k^4$

C $\frac{x^{-3}}{a^0y^5}$

$\frac{x^{-3}}{a^0y^5} = \frac{1}{x^3 \cdot 1 \cdot y^5}$ $a^0 = 1$ and $x^{-3} = \frac{1}{x^3}$.

$= \frac{1}{x^3y^5}$

CHECK IT OUT! **Simplify.**

4a. $2r^0m^{-3}$ **4b.** $\frac{r^{-3}}{7}$ **4c.** $\frac{g^4}{h^{-6}}$

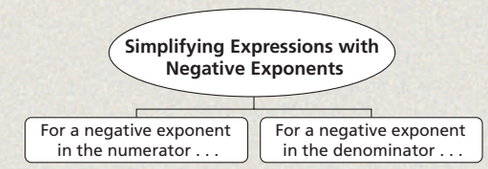
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Complete each equation: $2b^{?} = \frac{2}{b^2}$, $\frac{s^{-3}}{k^{?}} = \frac{1}{s^3}$, $?^{-2} = \frac{1}{t^2}$

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to simplify, and give an example.



MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- How do you decide which factors get moved to the other side of the fraction bar?
- What happens to factors with exponents of zero?

3 Close

Summarize

Remind students that an expression is not simplified if it has an exponent that is negative or zero.

Have students state the rules for simplifying expressions with negative exponents in their own words. Accept nontechnical answers such as the following: Make the exponent positive and move the factor to the other side of the fraction bar.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. -2; 0; t
2. See Additional Answers.

6-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

1. **Medicine** A typical virus is about 10^{-7} m in size. Simplify this expression. $\frac{1}{10{,}000{,}000}$ m

SEE EXAMPLE 2

Simplify.

2. 6^{-2} $\frac{1}{36}$
3. 3^{0} 1
4. -5^{-2} $-\frac{1}{25}$
5. 3^{-3} $\frac{1}{27}$
6. 1^{-8} 1
7. -8^{-3} $-\frac{1}{512}$
8. 10^{-2} $\frac{1}{100}$
9. $(4.2)^{0}$ 1
10. $(-3)^{-3}$ $-\frac{1}{27}$
11. 4^{-2} $\frac{1}{16}$

SEE EXAMPLE 3

Evaluate each expression for the given value(s) of the variable(s).

12. b^{-2} for $b = -3$ $\frac{1}{9}$
13. $(2t)^{-4}$ for $t = 2$ $\frac{1}{256}$
14. $(m-4)^{-5}$ for $m = 6$ $\frac{1}{32}$
15. $2x^{0}y^{-3}$ for $x = 7$ and $y = -4$ $-\frac{1}{32}$

SEE EXAMPLE 4

Simplify.

16. $4m^{0}$ 4
17. $3k^{-4}$ $\frac{3}{k^4}$
18. $\frac{7}{r^{-7}}$ $7r^7$
19. $\frac{x^{10}}{d^{-3}}$ $x^{10}d^3$
20. $2x^{0}y^{-4}$ $\frac{2}{y^4}$
21. $\frac{f^{-4}}{g^{-6}}$ $\frac{g^6}{f^4}$
22. $\frac{c^4}{d^{-3}}$ c^4d^3
23. $p^{7}q^{-1}$ $\frac{p^7}{q}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24	1
25–36	2
37–42	3
43–57	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

24. **Biology** One of the smallest bats is the northern blossom bat, which is found from Southeast Asia to Australia. This bat weighs about 2^{-1} ounce. Simplify this expression. $\frac{1}{2}$ oz

Simplify.

25. 8^{0} 1
26. 5^{-4} $\frac{1}{625}$
27. 3^{-4} $\frac{1}{81}$
28. -9^{-2} $-\frac{1}{81}$
29. -6^{-2} $-\frac{1}{36}$
30. 7^{-2} $\frac{1}{49}$
31. $\left(\frac{2}{5}\right)^{0}$ 1
32. 13^{-2} $\frac{1}{169}$
33. $(-3)^{-1}$ $-\frac{1}{3}$
34. $(-4)^{2}$ 16
35. $\left(\frac{1}{2}\right)^{-2}$ 4
36. -7^{-1} $-\frac{1}{7}$

Evaluate each expression for the given value(s) of the variable(s).

37. x^{-4} for $x = 4$ $\frac{1}{256}$
38. $\left(\frac{2}{3}v\right)^{-3}$ for $v = 9$ $\frac{1}{216}$
39. $(10-d)^{0}$ for $d = 11$ 1
40. $10m^{-1}n^{-5}$ for $m = 10$ and $n = -2$ $-\frac{1}{32}$
41. $(3ab)^{-2}$ for $a = \frac{1}{2}$ and $b = 8$ $\frac{1}{144}$
42. $4w^{v}x^{v}$ for $w = 3$, $v = 0$, and $x = -5$ 4

Simplify.

43. k^{-4} $\frac{1}{k^4}$
44. $2z^{-8}$ $\frac{2}{z^8}$
45. $\frac{1}{2b^{-3}}$ $\frac{b^3}{2}$
46. $c^{-2}d$ $\frac{d}{c^2}$
47. $-5x^{-3}$ $-\frac{5}{x^3}$
48. $4x^{-6}y^{-2}$ $\frac{4}{x^6y^2}$
49. $\frac{2f^{0}}{7g^{-10}}$ $\frac{2g^{10}}{7}$
50. $\frac{r^{-5}}{s^{-1}}$ $\frac{s}{r^5}$
51. $\frac{s^5}{t^{-12}}$ s^5t^{12}
52. $\frac{3w^{-5}}{x^{-6}}$ $\frac{3x^6}{w^5}$
53. $b^{0}c^{0}$ 1
54. $\frac{2}{3}m^{-1}n^{5}$ $\frac{2n^5}{3m}$
55. $\frac{q^{-2}r^{0}}{s^{0}}$ $\frac{1}{q^2}$
56. $\frac{a^{-7}b^{2}}{c^{3}d^{-4}}$ $\frac{b^2d^4}{a^7c^3}$
57. $\frac{h^{3}k^{-1}}{6m^{2}}$ $\frac{h^3}{6m^2k}$

B. G. Thomson/Photo Researchers, Inc.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2

Basic	24–36, 77, 86, 88
Average	24–36, 77, 86, 88
Advanced	24–36, 77, 100, 101

If you finished Examples 1–4

Basic	24–77, 86–99
Average	24–57, 58–74 even, 76–100
Advanced	24–57, 58–74 even, 77–101

Homework Quick Check

Quickly check key concepts.
Exercises: 24, 28, 34, 42, 52, 77

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 77, 94

Construct viable arguments and critique the reasoning of others. Exercises 66, 84–85

Look for and make use of structure. Exercise 101

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MULTI-STEP TEST PREP **Exercise 94** involves writing an equation with a negative exponent. This exercise prepares students for the Multi-Step Test Prep.

Answers

77. red blood cell: $\frac{1}{125{,}000}$ m; white blood cell: $\frac{3}{250{,}000}$ m; platelet: $\frac{3}{1{,}000{,}000}$ m

85. Possible answer: Look at the pattern below. As the exponent goes down by 1, the value is half of what it was before.

$2^3 = 8$, $2^2 = 4$,

$2^1 = 2$, $2^0 = 1$,

$x^{-x} = \frac{1}{2}$, $x^{-x} = \frac{1}{4}$,

$x^{-x} = \frac{1}{8} = \frac{1}{2^3}$

Evaluate each expression for $x = 3$, $y = -1$, and $z = 2$.

58. z^{-5} $\frac{1}{32}$ 59. $(x+y)^{-4}$ $\frac{1}{16}$ 60. $(yz)^0$ 1 61. $(xyz)^{-1}$ $-\frac{1}{6}$

62. $(xy-3)^{-2}$ $\frac{1}{36}$ 63. x^{-y} 3 64. $(yz)^{-x}$ $-\frac{1}{8}$ 65. xy^{-4} 3

66. ERROR ANALYSIS Look at the two equations below. Which is incorrect? Explain the error.

66. Equation A is incorrect because 5 was incorrectly moved to the denom. The neg. exp. applies only to the base *x*.

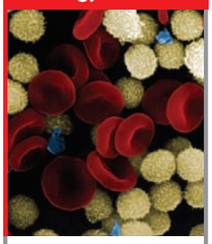
A $5x^{-3} = \frac{1}{5x^3}$ B $5x^{-3} = \frac{5}{x^3}$

Simplify.

67. a^3b^{-2} $\frac{a^3}{b^2}$ 68. $c^{-4}d^3$ $\frac{d^3}{c^4}$ 69. $v^0w^2y^{-1}$ $\frac{w^2}{y}$ 70. $(a^2b^{-7})^0$ 1 71. $-5y^{-6}$ $-\frac{5}{y^6}$

72. $\frac{2a^{-5}}{b^{-6}}$ $\frac{2b^6}{a^5}$ 73. $\frac{2a^3}{b^{-1}}$ $2a^3b$ 74. $\frac{m^2}{n^{-3}}$ m^2n^3 75. $\frac{x^{-8}}{3y^{12}}$ $\frac{1}{3x^8y^{12}}$ 76. $-\frac{20p^{-1}}{5q^{-3}}$ $-\frac{4q^3}{p}$

LINK Biology



When bleeding occurs, platelets (which appear blue in the image above) help to form a clot to reduce blood loss. Calcium and vitamin K are also necessary for clot formation.

77. **Biology** Human blood contains red blood cells, white blood cells, and platelets. The table shows the sizes of these components. Simplify each expression.

Blood Components	
Part	**Size (m)**
Red blood cell	$125{,}000^{-1}$
White blood cell	$3(500)^{-2}$
Platelet	$3(1000)^{-2}$

Tell whether each statement is sometimes, always, or never true.

78. If n is a positive integer, then $x^{-n} = \frac{1}{x^n}$. always

79. If x is positive, then x^{-n} is negative. never

80. If n is zero, then x^{-n} is 1. sometimes

81. If n is a negative integer, then $x^{-n} = 1$. sometimes

82. If x is zero, then x^{-n} is 1. never

83. If n is an integer, then $x^{-n} > 1$. sometimes

84. **Critical Thinking** Find the value of $2^3 \cdot 2^{-3}$. Then find the value of $3^2 \cdot 3^{-2}$. Make a conjecture about the value of $a^n \cdot a^{-n}$. 1; 1; $a^n \cdot a^{-n} = 1$

85. **Write About It** Explain in your own words why 2^{-3} is the same as $\frac{1}{2^3}$.

Find the missing value.

86. $\frac{1}{4} = 2^{\square}$ -2 87. $9^{-2} = \frac{1}{\square}$ 81 88. $\frac{1}{64} = \square^{-2}$ 8 89. $\frac{\square}{3} = 3^{-1}$ 1

90. $7^{-2} = \frac{1}{\square}$ 49 91. $10^{\square} = \frac{1}{1000}$ -3 92. $3 \cdot 4^{-2} = \frac{3}{\square}$ 16 93. $2 \cdot \frac{1}{5} = 2 \cdot 5^{\square}$ -1

MULTI-STEP TEST PREP

94. a. The product of the frequency f and the wavelength w of light in air is a constant v. Write an equation for this relationship. $fw = v$

b. Solve this equation for wavelength. Then write this equation as an equation with f raised to a negative exponent. b. $w = \frac{v}{f}$; $w = vf^{-1}$

c. The units for frequency are hertz (Hz). One hertz is one cycle per second, which is often written as $\frac{1}{s}$. Rewrite this expression using a negative exponent. s^{-1}

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PRACTICE A

PRACTICE C

PRACTICE B

Simplify.

1. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ 2. $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

3. $(-5)^{-2}$ $\frac{1}{25}$ 4. $-(4)^{-3}$ $-\frac{1}{64}$

5. -6^0 -1 6. $(7)^{-2}$ $\frac{1}{49}$

Evaluate each expression for the given value(s) of the variable(s).

7. d^{-5} for $d = -2$ $-\frac{1}{8}$ 8. a^2b^{-6} for $a = 3$ and $b = 2$ $\frac{243}{64}$ 9. $(b-4)^{-2}$ for $b = 1$ $\frac{1}{9}$

10. $5z^{-4}$ for $z = -3$ and $x = 2$ $\frac{5}{9}$ 11. $(5z)^{-4}$ for $z = -3$ and $x = 2$ $\frac{1}{225}$ 12. $c^{-3}(16^{-2})$ for $c = 4$ $\frac{1}{16{,}384}$

Simplify.

13. t^{-4} $\frac{1}{t^4}$ 14. $3r^{-5}$ $\frac{3}{r^5}$ 15. $\frac{s^{-2}}{t^{-5}}$ $\frac{t^5}{s^3}$

16. $\frac{h^0}{3}$ $\frac{1}{3}$ 17. $\frac{2x^{-3}y^{-2}}{z^4}$ $\frac{2}{x^3y^2z^4}$ 18. $\frac{4fg^{-5}}{5h^{-3}}$ $\frac{4fh^3}{5g^5}$

19. $\frac{14a^{-4}}{20bc^{-2}}$ $\frac{7c}{10a^4b}$ 20. $\frac{a^4c^2e^0}{b^{-3}d^{-3}}$ $a^4bc^2d^3$ 21. $\frac{-3g^{-2}hk^{-2}}{-6h^0}$ $\frac{h}{2g^2k^2}$

22. A cooking website claims to contain 10^5 recipes. Evaluate this expression. 100,000

23. A ball bearing has diameter 2^{-3} inches. Evaluate this expression. $\frac{1}{8}$ inch or 0.125 inch

READING STRATEGIES

Studying the patterns that are found in expressions with exponents can help you remember the rules for evaluating expressions with integer exponents.

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
$3^3 = 3 \cdot 3 \cdot 3 = 27$
$3^2 = 3 \cdot 3 = 9$
$3^1 = 3$
$3^0 = 1$
$3^{-1} = \frac{1}{3}$
$3^{-2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$
$3^{-3} = \frac{1}{3 \cdot 3 \cdot 3} = \frac{1}{27}$
$3^{-4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$

Positive exponents: The answer is the base multiplied by itself the number of times identified by the exponent.

Zero exponent: The answer is always 1 (if the base is not 0; 0^0 is undefined).

Negative exponents: The answer is the *reciprocal* of the same expression with a positive exponent.

Note that the rules are the same when the base is a variable:

$b^3 = b \cdot b \cdot b$ $g^0 = 1$ $k^{-5} = \frac{1}{k^5}$ $\frac{1}{m^{-3}} = m^3$

Answer each question.

1. What is the base of the expression 6^{-4}? 6
2. What number can go in the box to make a true statement: $5^{\square} = 1$? 0
3. Write the expression $\frac{1}{8^3}$ with a negative exponent. 8^{-3}
4. What is the *reciprocal* of b^7? $\frac{1}{b^7}$

Simplify each expression.

5. 2^5 32 6. 2^{-5} $\frac{1}{32}$
7. 7^0 1 8. 10^{-6} $\frac{1}{1{,}000{,}000}$
9. $(-4)^2$ 16 10. $(-4)^{-3}$ $-\frac{1}{64}$
11. t^{-4} $\frac{1}{t^4}$ 12. c^2d^{-3} $\frac{c^2}{d^3}$
13. $8x^{-5}$ $\frac{8}{x^5}$ 14. $12r^0$ 12

RETEACH

Remember that 2^3 means $2 \times 2 \times 2 = 8$. The base is 2, the exponent is positive 3. Exponents can also be 0 or negative.

	Zero Exponents	Negative Exponents	Negative Exponents in the Denominator
Definition	For any nonzero number x, $x^0 = 1$.	For any nonzero number x and any integer n, $x^{-n} = \frac{1}{x^n}$.	For any nonzero number x and any integer n, $\frac{1}{x^{-n}} = x^n$.
Examples	$6^0 = 1$ $\left(\frac{1}{2}\right)^0 = 1$	$5^{-3} = \frac{1}{5^3}$ $2^{-4} = \frac{1}{2^4}$	$\frac{1}{8^{-2}} = 8^2$ $\frac{1}{2^{-4}} = 2^4$

0^0 and 0^{-n} are undefined.

Simplify 4^{-2}.

4^{-2}
$\frac{1}{4^2}$ *Write without negative exponents.*
$\frac{1}{4 \cdot 4}$ *Write in expanded form.*
$\frac{1}{16}$ *Simplify.*

Simplify $x^2y^{-2}z^0$.

$x^2y^{-2}z^0$
$\frac{x^2z^0}{y^2}$ *Write without negative exponents.*
$\frac{x^2(1)}{y^2}$ $z^0 = 1$.
$\frac{x^2}{y^2}$ *Simplify.*

Fill in the blanks to simplify each expression.

1. 2^{-5}
$2^{-5} = \frac{1}{2^{\square}}$
$\frac{1}{2^5} = \frac{1}{\boxed{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}}$
$= \frac{1}{32}$

2. 10^{-3}
$10^{-3} = \frac{1}{10}$
$\frac{1}{10^3} = \frac{1}{\boxed{10 \cdot 10 \cdot 10}}$
$= \frac{1}{1000}$

3. $\frac{1}{5^{-4}}$
$\frac{1}{5^{-4}} = 5^{\square}$
$5^{\square} = \boxed{5 \cdot 5 \cdot 5 \cdot 5}$
$= 625$

Simplify.

4. $5y^{-4}$ $\frac{5}{y^4}$ 5. $\frac{8}{a^{-3}}$ $8a^3$ 6. $9x^3y^{-2}$ $\frac{9x^3}{y^2}$
7. $\frac{x^3}{x^{-5}y}$ $\frac{x^4}{y}$ 8. $\frac{b^2}{a^{-3}b^3}$ $\frac{a}{b}$ 9. $5x^{-4}y^2$ $\frac{5y^2}{x^4}$

TEST PREP

95. Which is NOT equivalent to the other three?

Ⓐ $\frac{1}{25}$ Ⓑ 5^{-2} Ⓒ 0.04 Ⓓ -25

96. Which is equal to 6^{-2}?

Ⓕ $6(-2)$ Ⓖ $(-6)(-6)$ Ⓗ $-\frac{1}{6 \cdot 6}$ Ⓙ $\frac{1}{6 \cdot 6}$

97. Simplify $\frac{a^3b^{-2}}{c^{-1}}$.

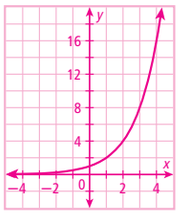
Ⓐ $\frac{a^3c}{b^2}$ Ⓑ $\frac{a^3b^2}{-c}$ Ⓒ $\frac{a^3}{-b^2c}$ Ⓓ $\frac{c}{a^3b^2}$

98. Gridded Response Simplify $\left[2^{-2} + (6+2)^0\right]$. $\frac{5}{4}$, or 1.25

99. Short Response If a and b are real numbers and n is a positive integer, write a simplified expression for the product $a^{-n} \cdot b^0$ that contains only positive exponents. Explain your answer. $\frac{1}{a^n}$; $a^{-n} = \frac{1}{a^n}$ and $b^0 = 1$ for $b \neq 0$. So you have $\frac{1}{a^n} \cdot 1$, or simply $\frac{1}{a^n}$.

CHALLENGE AND EXTEND

100.



100. Multi-Step Copy and complete the table of values below. Then graph the ordered pairs and describe the shape of the graph. Possible answer: y increases more rapidly as x increases.

x	−4	−3	−2	−1	0	1	2	3	4
$y = 2^x$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

101. Multi-Step Copy and complete the table. Then write a rule for the values of 1^n and $(-1)^n$ when n is any negative integer. $1^n = 1$; $(-1)^n = -1$ if n is odd, and $(-1)^n = 1$ if n is even.

n	−1	−2	−3	−4	−5
1^n	1	1	1	1	1
$(-1)^n$	−1	1	−1	1	−1

PROBLEM SOLVING

Write the correct answer.

1. At the 2005 World Exposition in Aichi, Japan, tiny mu-chips were embedded in the admissions tickets to prevent counterfeiting. The mu-chip was developed by Hitachi in 2003. Its area is $4^{(10)^{-3}}$ square millimeters. Simplify this expression. $\frac{4}{25}$ or 0.16 mm²

2. Despite their name, Northern Yellow Bats are commonly found in warm, humid areas in the southeast United States. An adult has a wingspan of about 14 inches and weighs between $3(2)^{-3}$ and $3(2)^{-2}$ ounces. Simplify these expressions. $\frac{3}{8}$ and $\frac{3}{4}$ oz

3. Saira is using the formula for the area of a circle to determine the value of π. She is using the expression Ar^{-2} where $A = 50.265$ and $r = 4$. Use a calculator to evaluate Saira's expression to find her approximation of the value of π to the nearest thousandth. 3.142

4. The volume of a freshwater tank can be expressed in terms of x, y, and z. Expressed in these terms, the volume of the tank is $x^2y^{-2}z$ liters. Determine the volume of the tank if $x = 4$, $y = 3$, and $z = 6$. $42\frac{2}{3}$ liters

Alison has an interest in entomology, the study of insects. Her collection of insects from around the world includes the four specimens shown in the table below. Select the best answer.

Insect	Mass
Emperor Scorpion	2^{-5} kg
African Goliath Beetle	11^{-1} kg
Giant Weta	2^{-4} kg
Madagascar Hissing Cockroach	5^{-3} kg

5. Cockroaches have been found on every continent, including Antarctica. What is the mass of Alison's Madagascar Hissing Cockroach expressed as a quotient?
A $-\frac{1}{125}$ kg C $\frac{1}{15}$ kg
B $\frac{1}{125}$ kg D 125 kg

6. Many Giant Wetas are so heavy that they cannot jump. Which expression is another way to show the mass of the specimen in Alison's collection?
F $-(2)4$ kg H $\frac{1}{2 \cdot 2 \cdot 2 \cdot 2}$ kg
G $\left(\frac{1}{2}\right)^{-4}$ kg J $4\frac{1}{2}$ kg

7. Scorpions are closely related to spiders and horseshoe crabs. What is the mass of Alison's Emperor Scorpion expressed as a quotient?
A $-\frac{1}{32}$ kg C $\frac{1}{32}$ kg
B $\frac{1}{25}$ kg D 32 kg

CHALLENGE

When you write out the first several powers of x^n, where x and n are positive integers, you can discover interesting patterns in the units digits of x^n.

	x^1	x^2	x^3	x^4	x^5	x^6
$x = 2$	$2^1 = 2$	$2^2 = 2(2) = 4$	$2^3 = 2(4) = 8$	$2^4 = 2(8) = 16$	$2^5 = 2(16) = 32$	$2^6 = 2(32) = 64$

Notice that 2^1 and 2^5 have the same units digit and that 2^2 and 2^6 have the same units digit. In the exercises that follow, you can discover other number patterns involving the units digits of x^n.

In Exercises 1–10, find the first nine powers of each value of x. Using the units digit of each result, complete the table. You may find a calculator useful.

		x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
1.	$x = 1$	1	1	1	1	1	1	1	1	1
2.	$x = 2$	2	4	8	6	2	4	8	6	2
3.	$x = 3$	3	9	7	1	3	9	7	1	3
4.	$x = 4$	4	6	4	6	4	6	4	6	4
5.	$x = 5$	5	5	5	5	5	5	5	5	5
6.	$x = 6$	6	6	6	6	6		6	6	6
7.	$x = 7$	7	9	3	1	7	9	3	1	7
8.	$x = 8$	8	4	2	6	8	4	2	6	8
9.	$x = 9$	9	1	9	1	9	1	9	1	9
10.	$x = 10$	0	0	0	0	0	0	0	0	0

Refer to the table that you completed in Exercises 1–10. Describe the pattern in the units digits of x^n.

11. 1^n For all n, 1^n has 1 as its units digit.
12. 2^n The pattern is 2, 4, 8, and 6, for $n = 1, 2, 3$, and 4 and then repeats.
13. 3^n The pattern is 3, 9, 7, and 1, for $n = 1, 2, 3$, and 4 and then repeats.
14. 5^n For all $n > 0$, 5^n has 5 as its units digit.
15. Write a rule that determines the units digit of 7^n as a function of n.
If you divide n by 4, then the units digit is 7, 9, 3, or 1, depending on whether the remainder is 1, 2, 3, or 0, respectively.

TEST PREP DOCTOR

In **Exercise 96,** if students chose **F,** they may have used the negative exponent as a factor. If they chose **G,** they may have made the base negative because the exponent is negative.

If students chose **D** in **Exercise 97,** they probably moved every factor to the opposite side of the fraction bar, even if it had a positive exponent.

Journal

Have students use patterns to explain why any number raised to the zero power, except zero, is one.

ALTERNATIVE ASSESSMENT

Have students choose three exercises from **Exercises 37–42,** write each expression in words, and then show two different ways to evaluate each expression.

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6-1 Lesson Quiz

1. A square foot is 3^{-2} square yards. Simplify this expression. $\frac{1}{9}$ yd²

Simplify.

2. 2^{-6} $\frac{1}{64}$

3. $(-7)^{-3}$ $-\frac{1}{343}$

4. 6^0 1 **5.** -11^2 −121

Evaluate each expression for the given value(s) of the variable(s).

6. x^{-4} for $x = 10$ $\frac{1}{10{,}000}$

7. $2a^{-1}b^{-3}$ for $a = 6$ and $b = 3$ $\frac{1}{81}$

Simplify.

8. $4y^{-5}$ $\frac{4}{y^5}$ **9.** $\frac{-3}{y^{-6}}$ $-3y^6$

10. $\frac{x^{-4}}{a^0y^3}$ $\frac{1}{x^4y^3}$

Also available online

6-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Evaluate and simplify expressions containing rational exponents.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify each expression.

1. $\sqrt{36}$ 6
2. $\sqrt[5]{0}$ 0
3. $\sqrt[3]{64}$ 4
4. $\sqrt[4]{1}$ 1
5. $\sqrt[5]{100{,}000}$ 10
6. $-\sqrt[3]{27}$ -3

Also available online

Math Humor

Q: What do $3\sqrt{7}$ and this textbook have in common?

A: Both have an index.

CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
CC.9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

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COMMON CORE

6-2 Rational Exponents

CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *Also* **CC.9-12.N.RN.2**

Objective
Evaluate and simplify expressions containing rational exponents.

Vocabulary
index

Why learn this?
You can use rational exponents to find the number of Calories animals need to consume each day to maintain health. (See Example 3.)

Recall that the radical symbol $\sqrt{\ }$ is used to indicate roots. The **index** is the small number to the left of the radical symbol that tells which root to take. For example, $\sqrt[3]{\ }$ represents a cube root. Since $2^3 = 2 \cdot 2 \cdot 2 = 8$, $\sqrt[3]{8} = 2$.

Another way to write nth roots is by using exponents that are fractions. For example, for $b > 1$, suppose $\sqrt{b} = b^k$.

$$\begin{aligned} \sqrt{b} &= b^k \\ \left(\sqrt{b}\right)^2 &= \left(b^k\right)^2 && \text{Square both sides.} \\ b^1 &= b^{2k} && \text{Power of a Power Property} \\ 1 &= 2k && \text{If } b^m = b^n, \text{ then } m = n. \\ \frac{1}{2} &= k && \text{Divide both sides by 2.} \end{aligned}$$

So for all $b > 1$, $\sqrt{b} = b^{\frac{1}{2}}$.

Helpful Hint
When $b = 0$, $\sqrt[n]{b} = 0$.
When $b = 1$, $\sqrt[n]{b} = 1$.

Know it! Note

Definition of $b^{\frac{1}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{1}{n}$ is equal to the nth root of that number.	$3^{\frac{1}{2}} = \sqrt{3}$ $5^{\frac{1}{4}} = \sqrt[4]{5}$ $2^{\frac{1}{7}} = \sqrt[7]{2}$	If $b > 1$ and n is an integer, where $n \geq 2$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$. $b^{\frac{1}{2}} = \sqrt{b}$, $b^{\frac{1}{3}} = \sqrt[3]{b}$, $b^{\frac{1}{4}} = \sqrt[4]{b}$, and so on.

EXAMPLE 1 **Simplifying $b^{\frac{1}{n}}$**

Simplify each expression.

A $125^{\frac{1}{3}}$

$125^{\frac{1}{3}} = \sqrt[3]{125} = \sqrt[3]{5^3}$ *Use the definition of $b^{\frac{1}{n}}$.*
$= 5$

B $64^{\frac{1}{6}} + 25^{\frac{1}{2}}$

$64^{\frac{1}{6}} + 25^{\frac{1}{2}} = \sqrt[6]{64} + \sqrt{25}$ *Use the definition of $b^{\frac{1}{n}}$.*
$= \sqrt[6]{2^6} + \sqrt{5^2}$
$= 2 + 5 = 7$

Remember!
$\sqrt{\ }$ is equivalent to $\sqrt[2]{\ }$.

1 Introduce

EXPLORATION

Rational Exponents

You will need a graphing calculator for this Exploration.

Recall that whole-number exponents mean repeated multiplication. For example, $3^4 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 81$. You use a radical to show the inverse operation: $\sqrt[4]{81} = 3$.

1. You can use your calculator to evaluate radicals. To find $\sqrt{4096}$, enter 3 and then press MATH. Select 5: $\sqrt{\ }$, enter 4096, and press ENTER.
2. Use your calculator to help you complete the table.
3. You can also use your calculator to explore fractional exponents. To evaluate $4096^{\frac{1}{3}}$, first enter 4096. Then press ^ (1 ÷ 3) and press ENTER.
4. Use your calculator to help you complete the table.

Radical	Value
$\sqrt{4096}$	64
$\sqrt[3]{4096}$	16
$\sqrt[4]{4096}$	8
$\sqrt[6]{4096}$	4

Power	Value
$4096^{\frac{2}{3}}$	64
$4096^{\frac{1}{3}}$	16
$4096^{\frac{1}{4}}$	8
$4096^{\frac{1}{6}}$	4

5. Based on the tables, $4096^{\frac{1}{3}} = \sqrt[3]{4096}$, $4096^{\frac{1}{4}} = \sqrt[4]{4096}$, $4096^{\frac{1}{6}} = \sqrt[6]{4096}$, $4096^{\frac{2}{3}} = \sqrt[3]{4096}$.
6. $4096^{\frac{1}{n}}$ is the nth root of 4096 ($4096^{\frac{1}{6}} = \sqrt[6]{4096}$).

THINK AND DISCUSS

5. **Describe** what you notice about the two tables.

Motivate

Write 4^2 and 4^3 on the board and have students simplify both expressions. 16 and 64

Then write $4^{2\frac{1}{2}}$ on the board. Ask students to guess the value of this expression and explain their thinking. Possible answer: 32; the value should be between 16 and 64.

Tell students that in this lesson they will learn why $4^{2\frac{1}{2}} = 32$.

Explorations and answers are provided in your online resources.

Simplify each expression.

1a. $81^{\frac{1}{4}}$ 3

1b. $121^{\frac{1}{2}} + 256^{\frac{1}{4}}$ 15

A fractional exponent can have a numerator other than 1, as in the expression $b^{\frac{2}{3}}$. You can write the exponent as a product in two different ways.

$$b^{\frac{2}{3}} = b^{\frac{1}{3}\cdot 2} \qquad b^{\frac{2}{3}} = b^{2\cdot\frac{1}{3}}$$
$$= \left(b^{\frac{1}{3}}\right)^2 \quad \textit{Power of a Power Property} \qquad = \left(b^2\right)^{\frac{1}{3}}$$
$$= \left(\sqrt[3]{b}\right)^2 \quad \textit{Definition of } b^{\frac{1}{n}} \qquad = \sqrt[3]{b^2}$$

Know it! Note

Definition of $b^{\frac{m}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{m}{n}$ is equal to the nth root of the number raised to the mth power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$	If $b > 1$ and m and n are integers, where $m \geq 1$ and $n \geq 2$, then $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$.

EXAMPLE 2 Simplifying Expressions with Fractional Exponents

Simplify each expression.

A $216^{\frac{2}{3}}$

$$216^{\frac{2}{3}} = \left(\sqrt[3]{216}\right)^2 \quad \textit{Definition of } b^{\frac{m}{n}}$$
$$= \left(\sqrt[3]{6^3}\right)^2$$
$$= (6)^2 = 36$$

B $32^{\frac{4}{5}}$

$$32^{\frac{4}{5}} = \left(\sqrt[5]{32}\right)^4$$
$$= \left(\sqrt[5]{2^5}\right)^4$$
$$= (2)^4 = 16$$

Simplify each expression.

2a. $16^{\frac{3}{4}}$ 8

2b. $1^{\frac{2}{5}}$ 1

2c. $27^{\frac{4}{3}}$ 81

EXAMPLE 3 *Biology Application*

The approximate number of Calories C that an animal needs each day is given by $C = 72m^{\frac{3}{4}}$, where m is the animal's mass in kilograms. Find the number of Calories that a 16 kg dog needs each day.

$$C = 72m^{\frac{3}{4}}$$
$$= 72(16)^{\frac{3}{4}} \quad \textit{Substitute 16 for } m.$$
$$= 72 \cdot \left(\sqrt[4]{16}\right)^3 \quad \textit{Definition of } b^{\frac{m}{n}}$$
$$= 72 \cdot \left(\sqrt[4]{2^4}\right)^3$$
$$= 72 \cdot (2)^3$$
$$= 72 \cdot 8 = 576$$

The dog needs 576 Calories per day to maintain health.

3. Find the number of Calories that an 81 kg panda needs each day. 1944

COMMON ERROR ALERT

With fractional exponents with a numerator other than 1, as in **Example 2,** students may confuse the index with the power. Write base $\overset{\text{power}}{\underset{\text{index}}{}}$ on the board for students to use as a reference.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Simplify each expression.

A. $343^{\frac{1}{3}}$ 7

B. $32^{\frac{1}{5}} + 9^{\frac{1}{2}}$ 5

Example 2

Simplify each expression.

A. $81^{\frac{5}{4}}$ 243

B. $3125^{\frac{2}{5}}$ 25

Example 3

Given a cube with surface area S, the volume V of the cube can be found by using the formula $V = \left(\frac{S}{6}\right)^{\frac{3}{2}}$. Find the volume of a cube with surface area 54 m². 27 m³

Also available online

2 Teach

Guided Instruction

Review powers and roots by writing $5^3 = 5 \cdot 5 \cdot 5 = 125$ and $\sqrt[3]{125} = 5$. Have students practice writing several of these examples. Then present the definition of $b^{\frac{1}{n}}$ and discuss **Example 1.** Show students two special cases: $1^{\frac{1}{n}} = 1$ and $0^{\frac{1}{n}} = 0$ for all natural-number values of n. Continue with the definition of $b^{\frac{m}{n}}$ and the remaining examples. Remind students of the properties of exponents before presenting **Example 4.**

Reaching All Learners *Through Cooperative Learning*

Have students work in pairs. Students take turns rolling both a red (r) and a blue (b) number cube. After each roll, the student uses the numbers shown on the cubes to complete the expression $64^{\frac{r}{b}}$. Then the student simplifies the expression or states that it cannot be simplified. The other student checks the answer and then rolls the number cubes to decide the next expression.

Mathematical Practices Questioning Strategies

EXAMPLE 1

- How do you change a fractional exponent to an nth root?

EXAMPLE 2

- Will you get the same answer if you raise the number to the power before taking the root?

EXAMPLE 3

- How can you use the order of operations to solve this problem?

Teaching Tip **Multiple Representations** In **Example 2,** the power can also be placed under the radical sign: $216^{\frac{2}{3}} = \sqrt[3]{216^2}$. However, it is usually more convenient to evaluate the root and then evaluate the power.

Power Presentations with PowerPoint®

Additional Examples

Example 4

Simplify. All variables represent nonnegative numbers.

A. $\sqrt[4]{a^4b^{20}}$ ab^5

B. $(x^6y^4)^{\frac{1}{2}}\sqrt{y^2}$ x^3y^3

Also available online

Questioning Strategies

EXAMPLE 4

- How do you use properties of exponents to simplify expressions?
- How can you check your answer after you have simplified an expression?

Answers to *Think and Discuss*

1. $\left(\sqrt[10]{25}\right)^5 = 25^{\frac{5}{10}} = 25^{\frac{1}{2}} = \sqrt{25} = 5$
2. See Additional Answers.

Remember that $\sqrt{\ }$ always indicates a nonnegative square root. When you simplify variable expressions that contain $\sqrt{\ }$, such as $\sqrt{x^2}$, the answer cannot be negative. But x may be negative. Therefore you simplify $\sqrt{x^2}$ as $|x|$ to ensure the answer is nonnegative.

When x is...	and n is...	x^n is...	and $\sqrt[n]{x^n}$ is...
Positive	Even	Positive	Positive
Negative	Even	Positive	Positive
Positive	Odd	Positive	Positive
Negative	Odd	Negative	Negative

When n is even, you must simplify $\sqrt[n]{x^n}$ to $|x|$, because you do not know whether x is positive or negative. When n is odd, simplify $\sqrt[n]{x^n}$ to x.

EXAMPLE 4 Using Properties of Exponents to Simplify Expressions

Simplify. All variables represent nonnegative numbers.

A $\sqrt[3]{x^9y^3}$

$\sqrt[3]{x^9y^3} = (x^9y^3)^{\frac{1}{3}}$ *Definition of* $b^{\frac{1}{n}}$

$= (x^9)^{\frac{1}{3}} \cdot (y^3)^{\frac{1}{3}}$ *Power of a Product Property*

$= \left(x^{9\cdot\frac{1}{3}}\right) \cdot \left(y^{3\cdot\frac{1}{3}}\right)$ *Power of a Power Property*

$= (x^3) \cdot (y^1) = x^3y$ *Simplify exponents.*

B $\left(x^2y^{\frac{1}{2}}\right)^4\sqrt[3]{y^3}$

$\left(x^2y^{\frac{1}{2}}\right)^4\sqrt[3]{y^3} = \left(x^2y^{\frac{1}{2}}\right)^4 \cdot y$ $\sqrt[3]{y^3} = y$

$= (x^{2\cdot4}) \cdot \left(y^{\frac{1}{2}\cdot4}\right) \cdot y$ *Power of a Product Property*

$= (x^8) \cdot (y^2) \cdot y$ *Simplify exponents.*

$= x^8 \cdot y^{2+1} = x^8y^3$ *Product of Powers Property*

Helpful Hint

When you are told that all variables represent nonnegative numbers, you do not need to use absolute values in your answers.

CHECK IT OUT! **Simplify. All variables represent nonnegative numbers.**

4a. $\sqrt[4]{x^4y^{12}}$

4b. $\dfrac{\left(xy^{\frac{1}{2}}\right)^2}{\sqrt[5]{x^5}}$

THINK AND DISCUSS

1. Explain how to find the value of $\left(\sqrt[10]{25}\right)^5$.

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, provide the definition and a numerical example of each type of exponent.

Exponent	Definition	Numerical Example
$b^{\frac{1}{n}}$		
$b^{\frac{m}{n}}$		

Know it! Note

3 Close

Summarize

Have students give an equivalent expression with fractional exponents for each expression below. Then have them simplify the expression.

1. $\sqrt[4]{81}$ $81^{\frac{1}{4}}$; 3
2. $\left(\sqrt{25}\right)^3$ $25^{\frac{3}{2}}$; 125
3. $\left(\sqrt[4]{256}\right)^3$ $256^{\frac{3}{4}}$; 64

Essential Question

Be sure students can answer the lesson's essential question:

How do you simplify an expression with a rational exponent? If the exponent has the form $\frac{1}{n}$, find the nth root of the base. If the exponent has the form $\frac{m}{n}$, find the nth root of the base raised to the mth power.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

6-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** In the expression $\sqrt[5]{3x}$, what is the *index*? 5

Simplify each expression.

SEE EXAMPLE 1

2. $8^{\frac{1}{3}}$ 2
3. $16^{\frac{1}{2}}$ 4
4. $0^{\frac{1}{6}}$ 0
5. $27^{\frac{1}{3}}$ 3
6. $81^{\frac{1}{2}}$ 9
7. $216^{\frac{1}{3}}$ 6
8. $1^{\frac{1}{9}}$ 1
9. $625^{\frac{1}{4}}$ 5
10. $36^{\frac{1}{2}} + 1^{\frac{1}{3}}$ 7
11. $8^{\frac{1}{3}} + 64^{\frac{1}{2}}$ 10
12. $81^{\frac{1}{4}} + 8^{\frac{1}{3}}$ 5
13. $25^{\frac{1}{2}} - 1^{\frac{1}{4}}$ 4

SEE EXAMPLE 2

14. $81^{\frac{3}{4}}$ 27
15. $8^{\frac{5}{3}}$ 32
16. $125^{\frac{2}{3}}$ 25
17. $25^{\frac{3}{2}}$ 125
18. $36^{\frac{3}{2}}$ 216
19. $64^{\frac{4}{3}}$ 256
20. $1^{\frac{3}{4}}$ 1
21. $0^{\frac{2}{3}}$ 0

SEE EXAMPLE 3

22. **Geometry** Given a square with area a, you can use the formula $P = 4a^{\frac{1}{2}}$ to find the perimeter P of the square. Find the perimeter of a square that has an area of 64 m^2. 32 m

SEE EXAMPLE 4

Simplify. All variables represent nonnegative numbers.

23. $\sqrt{x^4y^2}$ x^2y
24. $\sqrt[4]{z^4}$ z
25. $\sqrt{x^6y^6}$ x^3y^3
26. $\sqrt[3]{a^{12}b^6}$ a^4b^2
27. $\left(a^{\frac{1}{2}}\right)^2\sqrt{a^2}$ a^2
28. $\left(x^{\frac{1}{3}}\right)^6\sqrt[4]{y^4}$ x^2y
29. $\dfrac{\left(z^{\frac{1}{3}}\right)^3}{\sqrt{z^2}}$ 1
30. $\dfrac{\sqrt[3]{x^6y^9}}{x^2}$ y^3

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
31–42	1
43–50	2
51	3
52–59	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Simplify each expression.

31. $100^{\frac{1}{2}}$ 10
32. $1^{\frac{1}{5}}$ 1
33. $512^{\frac{1}{3}}$ 8
34. $729^{\frac{1}{2}}$ 27
35. $32^{\frac{1}{5}}$ 2
36. $196^{\frac{1}{2}}$ 14
37. $256^{\frac{1}{8}}$ 2
38. $400^{\frac{1}{2}}$ 20
39. $125^{\frac{1}{3}} + 81^{\frac{1}{2}}$ 14
40. $25^{\frac{1}{2}} - 81^{\frac{1}{4}}$ 2
41. $121^{\frac{1}{2}} - 243^{\frac{1}{5}}$ 8
42. $256^{\frac{1}{4}} + 0^{\frac{1}{3}}$ 4
43. $4^{\frac{3}{2}}$ 8
44. $27^{\frac{2}{3}}$ 9
45. $256^{\frac{3}{4}}$ 64
46. $64^{\frac{5}{6}}$ 32
47. $100^{\frac{3}{2}}$ 1000
48. $1^{\frac{5}{3}}$ 1
49. $9^{\frac{5}{2}}$ 243
50. $243^{\frac{2}{5}}$ 9

51. **Biology** Biologists use a formula to estimate the mass of a mammal's brain. For a mammal with a mass of m grams, the approximate mass B of the brain, also in grams, is given by $B = \frac{1}{8}m^{\frac{2}{3}}$. Find the approximate mass of the brain of a mouse that has a mass of 64 grams. 2g

Simplify. All variables represent nonnegative numbers.

52. $\sqrt[3]{a^6c^9}$ a^2c^3
53. $\sqrt[3]{8m^3}$ $2m$
54. $\sqrt[4]{x^{16}y^4}$ x^4y
55. $\sqrt[3]{27x^6}$ $3x^2$
56. $\left(x^{\frac{1}{2}}y^3\right)^2\sqrt{x^2}$ x^2y^6
57. $(a^2b^4)^{\frac{1}{2}}\sqrt[3]{b^6}$ ab^4
58. $\dfrac{\sqrt[3]{x^6y^6}}{yx^2}$ y
59. $\dfrac{\left(a^2b^{\frac{1}{2}}\right)^4}{\sqrt{b^2}}$ a^8b

Fill in the boxes to make each statement true.

60. $256^{\square} = 4$ 1
61. $\square^{\frac{1}{5}} = 1$ 1
62. $225^{\square} = 15$ 2
63. $\square^{\frac{1}{6}} = 0$ 0
64. $64^{\square} = 16$ 2
65. $\square^{\frac{3}{4}} = 125$ 625
66. $27^{\square} = 81$ 3
67. $36^{\square} = 216$ 3

6-2 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2
- **Basic** 31–50, 87, 88
- **Average** 32–50 even, 60–78 even
- **Advanced** 32–50 even, 60–78 even, 86–88, 91–93

If you finished Examples 1–4
- **Basic** 32–58 even, 60–80, 85, 87–90
- **Average** 32–58 even, 60–90, 92–96 even
- **Advanced** 32–80 even, 81–97

Homework Quick Check
Quickly check key concepts.
Exercises: 34, 40, 44, 51, 52, 56

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 22, 51, 80–81, 85

Reason abstractly and quantitatively. Exercise 97

Construct viable arguments and critique the reasoning of others. Exercises 82–84, 86

Look for and make use of structure. Exercises 60–67, 91–96

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 85** involves using rational exponents to model a real-world situation related to light. This exercise prepares students for the Multi-Step Test Prep.

Answers

86. $4^{\frac{3}{2}} = 4^{3 \cdot \frac{1}{2}} = (4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} = 8$; $4^{\frac{3}{2}} = 4^{\frac{1}{2} \cdot 3} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8$. It is often easier to take the square root first so that the remaining numbers in the calculation are smaller.

Simplify each expression.

68. $\left(\frac{81}{169}\right)^{\frac{1}{2}}$ $\frac{9}{13}$

69. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ $\frac{2}{3}$

70. $\left(\frac{256}{81}\right)^{\frac{1}{4}}$ $\frac{4}{3}$

71. $\left(\frac{1}{16}\right)^{\frac{1}{2}}$ $\frac{1}{4}$

72. $\left(\frac{9}{16}\right)^{\frac{3}{2}}$ $\frac{27}{64}$

73. $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ $\frac{4}{9}$

74. $\left(\frac{16}{81}\right)^{\frac{3}{4}}$ $\frac{8}{27}$

75. $\left(\frac{4}{49}\right)^{\frac{3}{2}}$ $\frac{8}{343}$

76. $\left(\frac{4}{25}\right)^{\frac{3}{2}}$ $\frac{8}{125}$

77. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$ $\frac{1}{27}$

78. $\left(\frac{27}{64}\right)^{\frac{2}{3}}$ $\frac{9}{16}$

79. $\left(\frac{8}{125}\right)^{\frac{4}{3}}$ $\frac{16}{625}$

80. Multi-Step Scientists have found that the life span of a mammal living in captivity is related to the mammal's mass. The life span in years L can be approximated by the formula $L = 12m^{\frac{1}{5}}$, where m is the mammal's mass in kilograms. How much longer is the life span of a lion compared with that of a wolf?

12 years

Typical Mass of Mammals	
Mammal	Mass (kg)
Koala	8
Wolf	32
Lion	243
Giraffe	1024

81. Geometry Given a sphere with volume V, the formula $r = 0.62V^{\frac{1}{3}}$ may be used to approximate the sphere's radius r. Find the approximate radius of a sphere that has a volume of 27 in^3. 1.86 in.

82. Critical Thinking Show that a number raised to the power $\frac{1}{3}$ is the same as the cube root of that number. (*Hint:* Use properties of exponents to find the cube of $b^{\frac{1}{3}}$. Then compare this with the cube of $\sqrt[3]{b}$. Use the fact that if two numbers have the same cube, then they are equal.)

82. $\left(b^{\frac{1}{3}}\right)^3 = b^{\frac{1}{3} \cdot 3} = b^1 = b$. Also, by definition $\left(\sqrt[3]{b}\right)^3 = b$. Therefore, $b^{\frac{1}{3}} = \sqrt[3]{b}$.

83. Critical Thinking Compare $n^{\frac{2}{3}}$ and $n^{\frac{3}{2}}$ for values of n greater than 1. When simplifying each of these expressions, will the result be greater than n or less than n? Explain.

83. $n^{\frac{2}{3}}$ will be less than n because $\frac{2}{3} < 1$. $n^{\frac{3}{2}}$ will be greater than n because $\frac{3}{2} > 1$.

84. ERROR ANALYSIS Two students simplified $64^{\frac{3}{2}}$. Which solution is incorrect? Explain the error.

A

$64^{\frac{3}{2}} = \left(\sqrt[3]{64}\right)^2$
$= (4)^2$
$= 16$

B

$64^{\frac{3}{2}} = \left(\sqrt{64}\right)^3$
$= (8)^3$
$= 512$

84. Solution A is incorrect. The first line should be $64^{\frac{3}{2}} = \left(\sqrt{64}\right)^3$.

MULTI-STEP TEST PREP

85. You can estimate an object's distance in inches from a light source by using the formula $d = \left(0.8\frac{L}{B}\right)^{\frac{1}{2}}$, where L is the light's luminosity in lumens and B is the light's brightness in lumens per square inch.

a. Find an object's distance to a light source with a luminosity of 4000 lumens and a brightness of 32 lumens per square inch. 10 in.

b. Suppose the brightness of this light source decreases to 8 lumens per square inch. How does the object's distance from the source change?

The distance doubles (20 in.)

PRACTICE A
PRACTICE C
PRACTICE B

Simplify each expression. All variables represent nonnegative numbers.

1. $27^{\frac{1}{3}}$ — 3
2. $121^{\frac{1}{2}}$ — 11
3. $0^{\frac{1}{3}}$ — 0
4. $64^{\frac{1}{2}} + 27^{\frac{1}{3}}$ — 11
5. $16^{\frac{1}{4}} + 8^{\frac{1}{3}}$ — 4
6. $100^{\frac{1}{2}} - 64^{\frac{1}{6}}$ — 8
7. $1^{\frac{1}{5}} + 49^{\frac{1}{2}}$ — 8
8. $25^{\frac{3}{2}}$ — 125
9. $32^{\frac{3}{5}}$ — 8
10. $16^{\frac{3}{4}}$ — 8
11. $1^{\frac{5}{6}}$ — 1
12. $121^{\frac{3}{2}}$ — 1331
13. $\sqrt[3]{y^5}$ — y
14. $\sqrt{x^4y^{12}}$ — x^2y^6
15. $\sqrt[3]{a^6b^3}$ — a^2b
16. $(x^{\frac{1}{2}})^4\sqrt{x^6}$ — x^5
17. $(x^{\frac{1}{2}}y)^3\sqrt{x^2y^2}$ — x^2y^4
18. $\frac{(x^{\frac{1}{2}}y^{\frac{1}{3}})^6}{\sqrt[3]{x^3}}$ — x

19. Given a cube with volume V, you can use the formula $P = 4V^{\frac{1}{3}}$ to find the perimeter of one of the cube's square faces. Find the perimeter of a face of a cube that has volume 125 m^3.
20 m

READING STRATEGIES

The table below summarizes the definitions you need when you work with fractional exponents.

Definition of $b^{\frac{1}{n}}$	A number raised to the power of $\frac{1}{n}$ is equal to the nth root of that number.	$9^{\frac{1}{2}} = \sqrt{9} = 3$ $32^{\frac{1}{5}} = \sqrt[5]{32} = 2$ $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$	In general, if $b > 1$ and n is an integer where $n \geq 2$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$.
Definition of $b^{\frac{m}{n}}$	A number raised to the power of $\frac{m}{n}$ is equal to the nth root of the number raised to the mth power.	$16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$ $100^{\frac{3}{2}} = \left(\sqrt{100}\right)^3 = 10^3 = 1000$	In general, if $b > 1$ and m and n are integers where $m \geq 1$ and $n \geq 2$, then $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$.

Complete each of the following.

1. The expression $27^{\frac{1}{3}}$ is equal to the 3rd or cube root of 27.
2. The expression $9^{\frac{5}{2}}$ is equal to the square root of 9 raised to the 5th power.
3. The expression $32^{\frac{4}{5}}$ is equal to the 5th root of 32 raised to the 4th power.

Simplify each expression.

4. $8^{\frac{1}{3}}$ — 2
5. $81^{\frac{1}{2}}$ — 9
6. $16^{\frac{1}{4}}$ — 2
7. $400^{\frac{1}{2}}$ — 20
8. $32^{\frac{2}{5}}$ — 4
9. $81^{\frac{3}{4}}$ — 27
10. $4^{\frac{3}{2}}$ — 8
11. $16^{\frac{5}{2}}$ — 1024
12. $256^{\frac{3}{4}}$ — 64

RETEACH

To simplify a number raised to the power of $\frac{1}{n}$, write the nth root of the number.

Simplify $216^{\frac{1}{3}}$.

$216^{\frac{1}{3}} = \sqrt[3]{216} = 6$

Think: What number, when taken as a factor 3 times, is equal to 216?
$6^3 = 6 \times 6 \times 6 = 216$, so $\sqrt[3]{216} = 6$.

When an expression contains two or more expressions with fractional exponents, evaluate the expressions with the exponents first, then add or subtract.

Simplify $81^{\frac{1}{2}} + 32^{\frac{1}{5}}$.

$81^{\frac{1}{2}} + 32^{\frac{1}{5}} = \sqrt{81} + \sqrt[5]{32}$
$= 9 + 2$
$= 11$

Simplify each expression.

1. $64^{\frac{1}{2}}$ — 8
2. $1000^{\frac{1}{3}}$ — 10
3. $1^{\frac{1}{5}}$ — 1
4. $256^{\frac{1}{4}}$ — 4
5. $32^{\frac{1}{5}}$ — 2
6. $49^{\frac{1}{2}}$ — 7
7. $8^{\frac{1}{3}} + 16^{\frac{1}{2}}$ — 6
8. $121^{\frac{1}{2}} + 27^{\frac{1}{3}}$ — 14
9. $32^{\frac{1}{5}} + 1^{\frac{1}{2}}$ — 3
10. $81^{\frac{1}{4}} - 16^{\frac{1}{2}}$ — 1
11. $144^{\frac{1}{2}} - 125^{\frac{1}{3}}$ — 7
12. $625^{\frac{1}{4}} - 0^{\frac{1}{2}}$ — 5

86. Write About It You can write $4^{\frac{3}{2}}$ as $4^{3\cdot\frac{1}{2}}$ or as $4^{\frac{1}{2}\cdot 3}$. Use the Power of a Power Property to show that both expressions are equal. Is one method easier than the other? Explain.

TEST PREP

87. What is $9^{\frac{1}{2}} + 8^{\frac{1}{3}}$?

Ⓐ 4 Ⓑ 5 Ⓒ 6 Ⓓ 10

88. Which expression is equal to 8?

Ⓕ $4^{\frac{3}{2}}$ Ⓖ $16^{\frac{1}{2}}$ Ⓗ $32^{\frac{4}{5}}$ Ⓙ $64^{\frac{3}{2}}$

89. Which expression is equivalent to $\sqrt[3]{a^9b^3}$?

Ⓐ a^2b Ⓑ a^3 Ⓒ a^3b Ⓓ a^3b^3

90. Which of the following is NOT equal to $16^{\frac{3}{2}}$?

Ⓕ $\left(\sqrt{16}\right)^3$ Ⓖ 4^3 Ⓗ $\left(\sqrt[3]{16}\right)^2$ Ⓙ $\sqrt{16^3}$

CHALLENGE AND EXTEND

Use properties of exponents to simplify each expression.

91. $\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)$ *a*

92. $\left(x^{\frac{1}{2}}\right)^5\left(x^{\frac{3}{2}}\right)$ x^4

93. $\left(x^{\frac{1}{3}}\right)^4\left(x^5\right)^{\frac{1}{3}}$ x^3

You can use properties of exponents to help you solve equations. For example, to solve $x^3 = 64$, raise both sides to the $\frac{1}{3}$ power to get $\left(x^3\right)^{\frac{1}{3}} = 64^{\frac{1}{3}}$. Simplifying both sides gives $x = 4$. Use this method to solve each equation. Check your answer.

94. $y^5 = 32$ 2

95. $27x^3 = 729$ 3

96. $1 = \frac{1}{8}x^3$ 2

97. Geometry The formula for the surface area of a sphere S in terms of its volume V is $S = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. What is the surface area of a sphere that has a volume of 36π cm^3? Leave the symbol π in your answer. What do you notice? 36π cm^2; both volume and surface area are described by 36π (although the units are different).

PROBLEM SOLVING

Write the correct answer.

1. For a pendulum with a length of L meters, the time in seconds that it takes the pendulum to swing back and forth is approximately $2L^{\frac{1}{2}}$. About how long does it take a pendulum that is 9 meters long to swing back and forth?
   6 s
2. The Beaufort Scale is used to measure the intensity of tornados. For a tornado with Beaufort number B, the formula $v = 1.9B^{\frac{3}{2}}$ may be used to estimate the tornado's wind speed in miles per hour. Estimate the wind speed of a tornado with Beaufort number 9.
   51.3 mi/h
3. Given a cube whose faces each have area A, the volume of the cube is given by the formula $V = A^{\frac{3}{2}}$. Find the volume of a cube whose faces each have an area of 64 in^2.
   512 in^3
4. At a factory that makes cylindrical cans, the formula $r = \left(\frac{V}{12}\right)^{\frac{1}{2}}$ is used to find the radius of a can with volume V. What is the radius of a can whose volume is 192 cm^3?
   4 cm

Given an animal's body mass m, in grams, the formula $B = 1.8m^{\frac{3}{4}}$ may be used to estimate the mass B, in grams, of the animal's brain. The table shows the body mass of several birds. Use the table for questions 5–7. Select the best answer.

5. Which is the best estimate for the brain mass of a macaw?
   A 9 g C 125 g
   B 45 g D 225 g
6. How much larger is the brain mass of a barn owl compared to the brain mass of a cockatiel?
   F 189 g H 388.8 g
   G 340.2 g J 1215 g
7. An animal has a body mass given by the expression x^4. Which expression can be used to estimate the animal's brain mass?
   A $B = 1.8x^3$ C $B = 1.8x^{12}$
   B $B = 1.8x^{\frac{3}{4}}$ D $B = 1.8x$

Typical Body Masses of Birds	
Bird	Body Mass (g)
Cockatiel	81
Guam Rail	256
Macaw	625
Barn Owl	1296

Sources:
http://www.beyondveg.com/billings-t/comp-anat/comp-anat-app2.shtml
http://www.sandiegozoo.org/animalbytes/index.html

CHALLENGE

Find a path from start to finish in the maze below. Each box that you pass through must have a value that is *greater than or equal to* the value in the previous box. You may only move horizontally or vertically to go from one box to the next.

START

$16^{\frac{1}{4}}$	$4^{\frac{1}{2}} - 27^{\frac{1}{3}}$	$0^{\frac{1}{5}}$	$1^{\frac{2}{3}} - 8^{\frac{1}{3}}$	$216^{\frac{2}{3}}$	$8^{\frac{1}{3}} + 9^{\frac{3}{2}}$
$36^{\frac{1}{2}} - 216^{\frac{1}{3}}$	$25^{\frac{1}{2}} - 32^{\frac{2}{5}}$	$9^{\frac{1}{2}} - 8^{\frac{1}{3}}$	$1^{\frac{3}{5}}$	$64^{\frac{1}{6}}$	$100^{\frac{1}{2}} + 27^{\frac{2}{3}}$
$1^{\frac{1}{2}} - 9^{\frac{3}{2}}$	$125^{\frac{2}{3}}$	$81^{\frac{3}{4}}$	$1000^{\frac{2}{3}}$	$16^{\frac{1}{2}} - 16^{\frac{1}{4}}$	$32^{\frac{1}{5}} + 0^{\frac{3}{4}}$
$512^{\frac{2}{9}}$	$9^{\frac{1}{2}} + 3^{\frac{4}{4}}$	$32^{\frac{2}{5}}$	$27^{\frac{1}{3}}$	$121^0 + 1^{\frac{1}{2}}$	$9^{\frac{1}{2}} - 3^0$
$49^{\frac{1}{2}} + 0^{\frac{1}{2}}$	$16^{\frac{3}{4}}$	$16^{\frac{1}{4}} - 1^{\frac{3}{4}}$	$1024^{\frac{1}{10}}$	$81^{\frac{1}{4}} + 49^{\frac{1}{2}}$	$243^{\frac{4}{5}}$
$16^{\frac{3}{4}} + 32^{\frac{1}{5}}$	$100^{\frac{1}{2}}$	$625^{\frac{1}{4}}$	$128^{\frac{2}{7}}$	$144^{\frac{1}{2}} - 81^{\frac{1}{2}}$	$64^{\frac{1}{3}}$
$125^{\frac{1}{3}} - 2^0$	$81^{\frac{3}{4}} - 32^{\frac{4}{5}}$	$256^{\frac{1}{2}}$	$16^{\frac{3}{4}} + 4^{\frac{3}{2}}$	$64^{\frac{2}{3}}$	$32^{\frac{1}{5}} + 100^{\frac{1}{2}}$
$625^{\frac{1}{4}}$	$64^{\frac{5}{6}} - 125^{\frac{2}{3}}$	$243^{\frac{2}{5}}$	$125^{\frac{2}{3}}$	$128^{\frac{3}{7}}$	$243^{\frac{1}{5}}$

FINISH

COMMON ERROR ALERT

In **Exercise 80,** students might multiply by 12 before evaluating the fifth root of m. Remind students of the order of operations: grouping symbols, exponents, multiply/divide, add/subtract. Remind them that radical signs are grouping symbols.

TEST PREP DOCTOR If students answer **B** for **Exercise 89,** they may think that $\sqrt[3]{b^3} = 1$ or $\left(b^3\right)^{\frac{1}{3}} = 1$. Remind them that cubing a number and taking the cube root are inverse operations that "undo" each other, so $\sqrt[3]{b^3} = \left(b^3\right)^{\frac{1}{3}} = b$.

Journal

Have students write the steps they would use to simplify $729^{\frac{5}{6}}$.

ALTERNATIVE ASSESSMENT

Have students write a quiz on rational exponents. The quiz should include five problems about simplifying expressions of various types. The quiz should also include answers.

Power Presentations with PowerPoint®

6-2 Lesson Quiz

Simplify each expression.

1. $16^{\frac{1}{4}}$ 2
2. $144^{\frac{1}{2}} - 27^{\frac{1}{3}}$ 9
3. $81^{\frac{3}{2}}$ 729
4. $64^{\frac{7}{6}}$ 128
5. In an experiment, the approximate population P of a bacteria colony is given by $P = 15t^{\frac{5}{3}}$, where t is the number of days since the start of the experiment. Find the population of the colony on the 8th day. 480

Simplify. All variables represent nonnegative numbers.

6. $\sqrt[5]{x^{10}z^5}$ x^2z
7. $\left(a^4b^{\frac{1}{4}}\right)^4\sqrt[3]{b^3}$ $a^{16}b^2$

Also available online

SECTION 6A

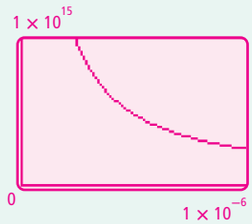
MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE Online Edition

Answers

2. 

As the wavelength increases, the frequency decreases.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

CHAPTER 6

MULTI-STEP TEST PREP

SECTION 6A

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

Exponents

I See the Light! The speed of light is the product of its frequency f and its wavelength w. In air, the speed of light is 3×10^8 m/s.

1. $f = \frac{3 \times 10^8}{w}$; $f = (3 \times 10^8)w^{-1}$

1. Write an equation for the relationship described above, and then solve this equation for frequency. Write this equation as an equation with w raised to a negative exponent.
2. Wavelengths of visible light range from 400 to 700 nanometers (10^{-9} meters). Use a graphing calculator and the relationship you found in Problem 1 to graph frequency as a function of wavelength. Sketch the graph with the axes clearly labeled. Describe your graph.
3. The speed of light in water is $\frac{3}{4}$ of its speed in air. Find the speed of light in water. 2.25×10^8 m/s
4. When light enters water, some colors bend more than others. How much the light bends depends on its wavelength. This is what creates a rainbow. The frequency of green light is about 5.9×10^{14} cycles per second. Find the wavelength of green light in water. about 3.81×10^{-7} m
5. When light enters water, colors with shorter wavelengths bend more than colors with longer wavelengths. Violet light has a frequency of 7.5×10^{14} cycles per second, and red light has a frequency of 4.6×10^{14} cycles per second. Which of these colors of light will bend more when it enters water? Justify your answer.

Violet light will bend more. Students can justify by using the inverse relationship, by using the graph, or by finding the wavelength for each color and comparing them.

Robert Fiusic/PhotoDisc/Getty Images

INTERVENTION

Scaffolding Questions

1. What does it mean to "solve for frequency"? Isolate the variable that represents frequency.
2. What are a reasonable domain and range? Possible answer: D: 380 to 760; R: 3×10^{14} to 9×10^{14}
3. How do you enter numbers in scientific notation into a calculator? Possible answer: 3×10^{14} is entered as 3 2nd EE , 14 or 3 × 10^x LOG 14.
4. What information do you need for this problem? speed of light in water Where can you find this information? answer to Problem 3
5. How does wavelength change as frequency changes? As one increases, the other decreases.

Extension

When you choose an FM radio station, you are choosing the frequency in MHz, or millions of waves per second (90 MHz = 90,000,000 waves per second). Find the wavelength for your favorite FM radio station. Possible answer: Using $w = (3 \times 10^8) f^{-1}$, FM 93.7 has an approximate wavelength of 3.2 m.

Quiz for Lessons 6-1 Through 6-2

6-1 Integer Exponents

Evaluate each expression for the given value(s) of the variable(s).

1. t^{-6} for $t = 2$ $\frac{1}{64}$
2. n^{-3} for $n = -5$ $-\frac{1}{125}$
3. $r^0 s^{-2}$ for $r = 8$ and $s = 10$ $\frac{1}{100}$

Simplify.

4. $5k^{-3}$ $\frac{5}{k^3}$
5. $\frac{x^4}{y^{-6}}$ x^4y^6
6. $8f^{-4}\,g^0$ $\frac{8}{f^4}$
7. $\frac{a^{-3}}{b^{-2}}$ $\frac{b^2}{a^3}$
8. **Measurement** Metric units can be written in terms of a base unit. The table shows some of these equivalencies. Simplify each expression.

Selected Metric Prefixes					
Milli-	Centi-	Deci-	Deka-	Hecto-	Kilo-
10^{-3}	10^{-2}	10^{-1}	10^{1}	10^{2}	10^{3}

6-2 Rational Exponents

Simplify each expression. All variables represent nonnegative numbers.

9. $81^{\frac{1}{2}}$ 9
10. $125^{\frac{1}{3}}$ 5
11. $4^{\frac{3}{2}}$ 8
12. $0^{\frac{2}{9}}$ 0
13. $\sqrt{x^8y^4}$ x^4y^2
14. $\sqrt[3]{r^9}$ r^3
15. $\sqrt[6]{z^{12}}$ z^2
16. $\sqrt[3]{p^3q^{12}}$ pq^4

READY TO GO ON?

SECTION 6A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

8. $10^{-3} = \frac{1}{1000}$, or 0.001;
$10^{-2} = \frac{1}{100}$, or 0.01;
$10^{-1} = \frac{1}{10}$, or 0.1;
$10^{1} = 10$; $10^{2} = 100$;
$10^{3} = 1000$

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 6A		
	Worksheets	Online	
Lesson 6-1	6-1 Intervention	Activity 6-1	Diagnose and Prescribe Online
Lesson 6-2	6-2 Intervention	Activity 6-2	

YES ENRICH

***Ready to Go On? Enrichment*, Section 6A**
Worksheets
Online

SECTION 6B

Polynomials

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 6-3 Polynomials • Classify polynomials and write polynomials in standard form. • Evaluate polynomial expressions. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** index cards, scissors, tape, ruler (MK), 8.5-by-11 inch paper
Algebra Lab Model Polynomial Addition and Subtraction • Use algebra tiles to model polynomial addition and subtraction. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 6-4 Adding and Subtracting Polynomials • Add and subtract polynomials. ☑ SAT-10 ☑ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Technology Lab	**Optional** books, pencils, slips of paper
Algebra Lab Model Polynomial Multiplication • Use algebra tiles to model polynomial multiplication. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 6-5 Multiplying Polynomials • Multiply polynomials. ☐ SAT-10 ☑ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests		
Extension Closure • Identify sets and the operations under which they are closed. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		
Lesson 6-6 Special Products of Binomials • Find special products of binomials ☐ SAT-10 ☑ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63a, 63b, 63c, 63d, 65, 75, 77, 79a, 79b

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 55, 57, 57, 69, 71

Lesson 5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25a, 25b, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65a, 65b, 65c, 65d, 67, 69a, 69b, 71, 73, 75, 77, 79, 81, 87, 89, 91, 93, 95, 97

Lesson 6: 3, 5, 7, 9, 11, 13, 15, 17, 19, 31, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 65, 67, 69, 71, 73

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

TERMINOLOGY

Lesson 6-3

In order to discuss polynomials, we must agree on terminology. The basic unit is the *monomial.* A monomial is a product of a real number and one or more variables with whole-number exponents. (The real number is usually rational, particularly within the scope of Algebra 1, but this is not a requirement.)
A *polynomial* is a sum of monomials. For example, the polynomial $8x^4 - 3x - 1$ may be written as $8x^4 + (-3x) + (-1)$, which is the sum of the monomials $8x^4$, $-3x$, and -1.

POLYNOMIALS

Lesson 6-3

Polynomials are in many ways analogous to counting numbers. Because our number system is base 10, all counting numbers can be written in expanded form in terms of powers of 10. For example, consider the expanded form of 653.

$$\begin{aligned} 653 &= 6 \cdot 100 + 5 \cdot 10 + 3 \cdot 1 \\ &= 6 \cdot 10^2 + 5 \cdot 10^1 + 3 \cdot 10^0 \end{aligned}$$

You can create a polynomial by replacing each of the 10s by a variable, such as x.

$$\begin{array}{ccccc} 6 \cdot 10^2 & + & 5 \cdot 10^1 & + & 3 \cdot 10^0 \\ \downarrow & & \downarrow & & \downarrow \\ 6 \cdot x^2 & + & 5 \cdot x^1 & + & 3 \cdot x^0 \end{array}$$

This polynomial is usually written in the more familiar form $6x^2 + 5x + 3$. For counting numbers, the only permissible multipliers of the powers of 10 are the digits 0 through 9, inclusive. For polynomials, any real number can be a multiplier of the variable terms.
The goal of this analogy is not to suggest that there is a correspondence between counting numbers and polynomials but to demonstrate that diverse mathematical concepts sometimes share underlying structures. As such, it makes sense to pose some of the same questions about polynomials that one might pose about counting numbers. For example, can we add, subtract, multiply, and divide polynomials? How?

POLYNOMIAL OPERATIONS

Lesson 6-4 to 6-6

Adding and subtracting polynomials is fairly straightforward because the process is nothing more than combining like terms.

$$(3x^2 + 7x + 5) + (2x + 6) = 3x^2 + 9x + 11$$

Polynomial multiplication can present greater difficulty for students, so it is essential to build gradually. Multiplication of two monomials is a natural starting point. You can use the Commutative and Associative Properties to show that $(4x)(2x) = 8x^2$.

Use the Distributive Property when multiplying a monomial and a binomial:

$$5x(2x + 3) = (5x)(2x) + (5x)(3) = 10x^2 + 15x$$

The Distributive Property is used repeatedly when multiplying a binomial by a binomial:

$$\begin{aligned} (3x + 2)(7x + 4) &= (3x)(7x + 4) + (2)(7x + 4) \\ &= 21x^2 + 12x + 14x + 8 \\ &= 21x^2 + 26x + 8 \end{aligned}$$

In fact, the Distributive Property can be used to multiply any two polynomials, regardless of the number of terms. The product will have one term for each product of a term from the first polynomial and a term from the second polynomial. So, the product of a binomial (2 terms) and a trinomial (3 terms) will have $2 \cdot 3 = 6$ terms before simplifying:

$$\begin{aligned} &(x + 2)(x^2 + 6x + 8) \\ &= (x + 2)(x^2) + (x + 2)(6x) + (x + 2)(8) \\ &= (x)(x^2) + 2(x^2) + x(6x) + 2(6x) + x(8) + 2(8) \end{aligned}$$

It is important to remember that this rule is true before the product is simplified. Clearly, some of the terms above are like terms and will be combined; the final answer will have fewer than 6 terms. In general, the product of a polynomial with m terms and a polynomial with n terms has mn terms before simplifying.

6-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Classify polynomials and write polynomials in standard form.

Evaluate polynomial expressions.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression for the given value of *x*.

1. $2x + 3; x = 2$ 7
2. $x^2 + 4; x = -3$ 13
3. $-4x - 2; x = -1$ 2
4. $7x^2 + 2x; x = 3$ 69

Identify the coefficient in each term.

5. $4x^3$ 4
6. y^3 1
7. $2n^7$ 2
8. $-s^4$ −1

Also available online

Math Humor

Q: What happened to the quadratic polynomial when he fell asleep on the beach?

A: He got second-degree burns.

COMMON CORE

CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*
a. Interpret parts of an expression, such as terms, factors, and coefficients.

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COMMON CORE

6-3 Polynomials

CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

Objectives
Classify polynomials and write polynomials in standard form.

Evaluate polynomial expressions.

Vocabulary
monomial
degree of a monomial
polynomial
degree of a polynomial
standard form of a polynomial
leading coefficient
quadratic
cubic
binomial
trinomial

Who uses this?

Pyrotechnicians can use polynomials to plan complex fireworks displays. (See Example 5.)

A **monomial** is a number, a variable, or a product of numbers and variables with whole-number exponents.

Monomials	Not Monomials
$5 \quad x \quad -7xy \quad 0.5x^4$	$-0.3x^{-2} \quad 4x - y \quad \frac{2}{x^3}$

The **degree of a monomial** is the sum of the exponents of the variables. A constant has degree 0.

EXAMPLE 1 Finding the Degree of a Monomial

Find the degree of each monomial.

A $-2a^2b^4$
The degree is 6. *Add the exponents of the variables: 2 + 4 = 6*

B 4
$4x^0$ *There is no variable, but you can write 4 as $4x^0$.*
The degree is 0.

C $8y$
$8y^1$ *A variable written without an exponent has exponent 1.*
The degree is 1.

Remember!
The *terms* of an expression are the parts being added or subtracted.

CHECK IT OUT! **Find the degree of each monomial.**
1a. $1.5k^2m$ **1b.** $4x$ **1c.** $2c^3$

A **polynomial** is a monomial or a sum or difference of monomials. The **degree of a polynomial** is the degree of the term with the greatest degree.

EXAMPLE 2 Finding the Degree of a Polynomial

Find the degree of each polynomial.

A $4x - 18x^5$
$4x$: degree 1 $-18x^5$: degree 5 *Find the degree of each term.*

The degree of the polynomial is the greatest degree, 5.

© Jeff Hunter/Getty Images

1 Introduce

EXPLORATION

Polynomials

1. The table shows examples of expressions that are and are not *polynomials*. What are some characteristics that the polynomials have in common? Describe some ways in which the polynomials are different from the expressions that are not polynomials.

Polynomials	Not Polynomials
$4x^2 - 5x + 1$	3^x
$3y^8 - 0.5y$	$0.6x^{-1} + 7$
$\frac{1}{2}z^5 - 2z^4 + 8z^2 + \frac{3}{4}z + \frac{1}{3}$	$\frac{4}{x^2}$
$-9x^2$	$-2z^3 + 3z^2 + \frac{1}{z}$
$1.7xy$	$3xy^{-5} + 3x^2y^{-8}$
$4x^2y - x^3y^2$	$\frac{1}{xy^2} + 2$

2. What do you notice about the exponents in the polynomials?
3. Do you think $16x^2 + 2xy + 8y^{-2}$ is a polynomial? Why or why not?

THINK AND DISCUSS

4. **Show** your own examples of expressions that are and are not polynomials.
5. **Describe** how you can tell whether an expression is a polynomial.

1. Polynomials all consist of single terms or terms being added or subtracted; in polynomials, variables do not appear in exponents or denominators; exponents in polynomials are all positive.

Motivate

Write $m^3 + n^2 + p + 5$ on the board. Ask students how many terms are in the expression. 4

Then have students identify the exponent of each variable.

m: 3

n: 2

p: 1

Ask students to identify the greatest exponent. 3

Explorations and answers are provided in your online resources.

Find the degree of each polynomial.

B $0.5x^2y + 0.25xy + 0.75$

$0.5x^2y$: degree 3 $0.25xy$: degree 2 0.75: degree 0

The degree of the polynomial is the greatest degree, 3.

C $6x^4 + 9x^2 - x + 3$

$6x^4$: degree 4 $9x^2$: degree 2 $-x$: degree 1 3: degree 0

The degree of the polynomial is the greatest degree, 4.

CHECK IT OUT! **Find the degree of each polynomial.**

2a. $5x - 6$ 1 **2b.** $x^3y^2 + x^2y^3 - x^4 + 2$ 5

The terms of a polynomial may be written in any order. However, polynomials that contain only one variable are usually written in *standard form*.

The **standard form of a polynomial** that contains one variable is written with the terms in order from greatest degree to least degree. When written in standard form, the coefficient of the first term is called the **leading coefficient**.

EXAMPLE 3 Writing Polynomials in Standard Form

Write each polynomial in standard form. Then give the leading coefficient.

A $20x - 4x^3 + 2 - x^2$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{20x}_{1} \; \underbrace{-4x^3}_{3} \; \underbrace{+2}_{0} \; \underbrace{-x^2}_{2} \rightarrow \underbrace{-4x^3}_{3} \; \underbrace{-x^2}_{2} \; \underbrace{+20x}_{1} \; \underbrace{+2}_{0}$$

The standard form is $-4x^3 - x^2 + 20x + 2$. The leading coefficient is -4.

B $y^3 + y^5 + 4y$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{y^3}_{3} \; \underbrace{+y^5}_{5} \; \underbrace{+4y}_{1} \rightarrow \underbrace{y^5}_{5} \; \underbrace{+y^3}_{3} \; \underbrace{+4y}_{1}$$

The standard form is $y^5 + y^3 + 4y$. The leading coefficient is 1.

Remember!

A variable written without a coefficient has a coefficient of 1.

$y^5 = 1y^5$

CHECK IT OUT! **Write each polynomial in standard form. Then give the leading coefficient.**

3a. $16 - 4x^2 + x^5 + 9x^3$ $x^5 + 9x^3 - 4x^2 + 16$; 1

3b. $18y^5 - 3y^8 + 14y$ $-3y^8 + 18y^5 + 14y$; -3

Some polynomials have special names based on their degree and the number of terms they have.

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic
6 or more	6th degree, 7th degree, and so on

Terms	Name
1	Monomial
2	Binomial
3	Trinomial
4 or more	Polynomial

2 Teach

Guided Instruction

Discuss with students examples and non-examples of monomials. Explain how to find the degree of a monomial and the degree of a polynomial. Be sure students get plenty of exposure to the new vocabulary in this lesson—it will be used often throughout the remainder of this chapter and the *Factoring Polynomials* chapter.

Reaching All Learners

Through Kinesthetic Experience

Divide students into groups of five, and have each write a monomial on an index card in large print. They may write a constant or a monomial with a variable. Tell them the variable must be *x*, but it can be raised to any power between 1 and 4, and can have any coefficient. Then call out classifications such as *quadratic trinomial* and *quartic binomial*. Have students in each group stand and arrange themselves to form that polynomial, holding their monomial in front of them for the rest of the class to see.

COMMON ERROR ALERT

Students often confuse the degree of a polynomial with the number of terms. Students may be less likely to confuse the two if they equate the word *degree* with an exponent-related phrase such as "maximum power."

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Additional Examples

Example 1

Find the degree of each monomial.

A. $4p^4q^3$ 7

B. $7ed$ 2

C. 3 0

Example 2

Find the degree of each polynomial.

A. $11x^7 + 3x^3$ 7

B. $\frac{1}{3}w^2z + \frac{1}{2}z^4 - 5$ 4

Example 3

Write each polynomial in standard form. Then give the leading coefficient.

A. $6x - 7x^5 + 4x^2 + 9$

$-7x^5 + 4x^2 + 6x + 9$; -7

B. $y^2 + y^6 - 3y$

$y^6 + y^2 - 3y$; 1

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- If a monomial has more than one variable with an exponent, how do you determine its degree?
- Why is the degree of a constant always zero?

EXAMPLE 2

- Do coefficients affect the degree of the polynomial? Explain.

EXAMPLE 3

- When is the coefficient of the first term also the leading coefficient of the polynomial?

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Additional Examples

Example 4

Classify each polynomial according to its degree and number of terms.

A. $5n^3 + 4n$ cubic binomial

B. $4y^6 - 5y^3 + 2y - 9$ 6th-degree polynomial

C. $-2x$ linear monomial

Example 5

A tourist accidentally drops her lip balm off the Golden Gate Bridge. The bridge is 220 feet from the water of the bay. The height of the lip balm is given by the polynomial $-16t^2 + 220$, where t is time in seconds. How far above the water will the lip balm be after 3 seconds? 76 ft

Also available online

Questioning Strategies

EXAMPLE 4

- Why will a constant polynomial always be a monomial?
- How does writing a polynomial in standard form help you classify the polynomial?

EXAMPLE 5

- What step in the order of operations will never be used when evaluating a polynomial in standard form? Why?

EXAMPLE 4 Classifying Polynomials

Classify each polynomial according to its degree and number of terms.

A $5x - 6$

Degree: 1 Terms: 2 $5x - 6$ is a linear binomial.

B $y^2 + y + 4$

Degree: 2 Terms: 3 $y^2 + y + 4$ is a quadratic trinomial.

C $6x^7 + 9x^2 - x + 3$

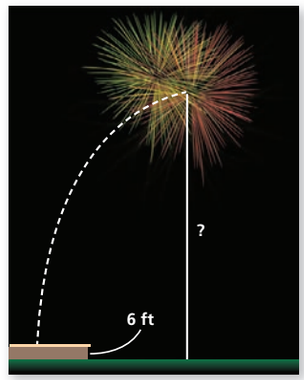
Degree: 7 Terms: 4 $6x^7 + 9x^2 - x + 3$ is a 7th-degree polynomial.

CHECK IT OUT! **Classify each polynomial according to its degree and number of terms.**

4a. $x^3 + x^2 - x + 2$ **4b.** 6 **4c.** $-3y^8 + 18y^5 + 14y$

EXAMPLE 5 *Physics Application*

A firework is launched from a platform 6 feet above the ground at a speed of 200 feet per second. The firework has a 5-second fuse. The height of the firework in feet is given by the polynomial $-16t^2 + 200t + 6$, where t is the time in seconds. How high will the firework be when it explodes?



Substitute the time for t to find the firework's height.

$-16t^2 + 200t + 6$

$-16(5)^2 + 200(5) + 6$ *The time is 5 seconds.*

$-16(25) + 200(5) + 6$

$-400 + 1000 + 6$ *Evaluate the polynomial by using the order of operations.*

606

When the firework explodes, it will be 606 feet above the ground.

CHECK IT OUT! **5. What if...?** Another firework with a 5-second fuse is launched from the same platform at a speed of 400 feet per second. Its height is given by $-16t^2 + 400t + 6$. How high will this firework be when it explodes?

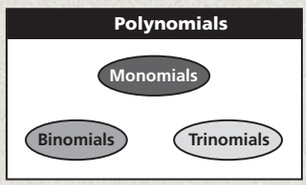
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain why each expression is not a polynomial: $2x^2 + 3x^{-3}$; $1 - \frac{a}{b}$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each oval, write an example of the given type of polynomial.



3 Close

Summarize

Have students give examples of monomials, binomials, trinomials, and polynomials of fourth and fifth degree. Write their examples on the board as they say them. For each, have students state whether the polynomial is in standard form, and if not, rewrite it in standard form.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: $2x^2 + 3x^{-3}$ contains an expression with a negative exponent. $1 - \frac{a}{b}$ contains a variable within a denominator.
2. See Additional Answers.

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Parent Resources Online

GUIDED PRACTICE

Vocabulary Match each polynomial on the left with its classification on the right.

1. $2x^3 + 6$ **d**
2. $3x^3 + 4x^2 - 7$ **c**
3. $5x^2 - 2x + 3x^4 - 6$ **a**

a. quartic polynomial
b. quadratic polynomial
c. cubic trinomial
d. cubic binomial

SEE EXAMPLE 1 Find the degree of each monomial.

4. 10^6 **0**
5. $-7xy^2$ **3**
6. $0.4n^8$ **8**
7. 2 **0**

SEE EXAMPLE 2 Find the degree of each polynomial.

8. $x^2 - 2x + 1$ **2**
9. $0.75a^2b - 2a^3b^5$ **8**
10. $15y - 84y^3 + 100 - 3y^2$ **3**
11. $r^3 + r^2 - 5$ **3**
12. $a^3 + a^2 - 2a$ **3**
13. $3k^4 + k^3 - 2k^2 + k$ **4**

SEE EXAMPLE 3 Write each polynomial in standard form. Then give the leading coefficient.

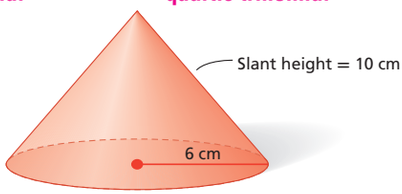
14. $-2b + 5 + b^2$ **$b^2 - 2b + 5$; 1**
15. $9a^8 - 8a^9$ **$-8a^9 + 9a^8$; −8**
16. $5s^2 - 3s + 3 - s^7$ **$-s^7 + 5s^2 - 3s + 3$; −1**
17. $2x + 3x^2 - 1$ **$3x^2 + 2x - 1$; 3**
18. $5g - 7 + g^2$ **$g^2 + 5g - 7$; 1**
19. $3c^2 + 5c^4 + 5c^3 - 4$ **$5c^4 + 5c^3 + 3c^2 - 4$; 5**

SEE EXAMPLE 4 Classify each polynomial according to its degree and number of terms.

20. $x^2 + 2x + 3$ **quadratic trinomial**
21. $x - 7$ **linear binomial**
22. $8 + k + 5k^4$ **quartic trinomial**
23. $q^2 + 6 - q^3 + 3q^4$ **quartic polynomial**
24. $5k^2 + 7k^3$ **cubic binomial**
25. $2a^3 + 4a^2 - a^4$ **quartic trinomial**

SEE EXAMPLE 5

26. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of this cone. **301.44 cm²**



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–34	1
35–40	2
41–49	3
50–57	4
58	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Find the degree of each monomial.

27. $3y^4$ **4**
28. $6k$ **1**
29. $2a^3b^2c$ **6**
30. 325 **0**
31. $2y^4z^3$ **7**
32. $9m^5$ **5**
33. p **1**
34. 5 **0**

Find the degree of each polynomial.

35. $a^2 + a^4 - 6a$ **4**
36. $3^2b - 5$ **1**
37. $3.5y^2 - 4.1y - 6$ **2**
38. $-5f^4 + 2f^6 + 10f^8$ **8**
39. $4n^3 - 2n$ **3**
40. $4r^3 + 4r^6$ **6**

Write each polynomial in standard form. Then give the leading coefficient.

41. $2.5 + 4.9t^3 - 4t^2 + t$ **$4.9t^3 - 4t^2 + t + 2.5$; 4.9**
42. $8a - 10a^2 + 2$ **$-10a^2 + 8a + 2$; −10**
43. $x^7 - x + x^3 - x^5 + x^{10}$ **$x^{10} + x^7 - x^5 + x^3 - x$; 1**
44. $-m + 7 - 3m^2$ **$-3m^2 - m + 7$; −3**
45. $3x^2 + 5x - 4 + 5x^3$ **$5x^3 + 3x^2 + 5x - 4$; 5**
46. $-2n + 1 - n^2$ **$-n^2 - 2n + 1$; −1**
47. $4d + 3d^2 - d^3 + 5$ **$-d^3 + 3d^2 + 4d + 5$; −1**
48. $3s^2 + 12s^3 + 6$ **$12s^3 + 3s^2 + 6$; 12**
49. $4x^2 - x^5 - x^3 + 1$ **$-x^5 - x^3 + 4x^2 + 1$; −1**

Teacher to Teacher

For many students it is easier to identify the degree of each term in a polynomial when each term is written with the variables and their exponents. For example:

$5x^2y + 25xy + 75$
$= 5x^2y^1 + 25x^1y^1 + 75x^0y^0$
$5x^2y^1 \longrightarrow$ degree $2 + 1 = 3$
$25x^1y^1 \longrightarrow$ degree $1 + 1 = 2$
$75x^0y^0 \longrightarrow$ degree $0 + 0 = 0$

The highest degree is 3, so the polynomial $5x^2y + 25xy + 75$ has degree 3.

Arlane Frederick
Buffalo, NY

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
Basic 27–49
Average 27–49
Advanced 27–49

If you finished Examples **1–5**
Basic 27–78
Average 27–79
Advanced 27–74, 76–80

Homework Quick Check
Quickly check key concepts.
Exercises: 27, 32, 34, 42, 52, 58, 60

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 26, 58, 63, 74, 79

Reason abstractly and quantitatively. Exercises 59–62

Construct viable arguments and critique the reasoning of others. Exercises 73, 75

Look for and make use of structure. Exercise 80

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State Resources Online

MULTI-STEP TEST PREP **Exercise 74** involves expressing the area and perimeter of a rectangle as polynomials. This exercise prepares students for the Multi-Step Test Prep.

Teaching Tip **Kinesthetic** Have students make (or attempt to make) the boxes for parts **a, b,** and **c** of **Exercise 63.** Give students scissors and tape and make sure they have a ruler and 8.5-by-11-inch paper (standard size of notebook or copy paper). The ruler can be found in the Manipulatives Kit (MK).

Answers

78.

Time (s)	Height (ft)
1	59
2	86
3	81
4	44

The rocket will be highest after 2 s.

LINK Transportation

Hybrid III is the crash test dummy used by the Insurance Institute for Highway Safety. During a crash test, sensors in the dummy's head, neck, chest, legs, and feet measure and record forces. Engineers study this data to help design safer cars.

Classify each polynomial according to its degree and number of terms.

50. 12 constant monomial
51. $6k$ linear monomial
52. $3.5x^3 - 4.1x - 6$ cubic trinomial
53. $4g + 2g^2 - 3$ quadratic trinomial
54. $2x^2 - 6x$ quadratic binomial
55. $6 - s^3 - 3s^4$ quartic trinomial
56. $c^2 + 7 - 2c^3$ cubic trinomial
57. $-y^2$ quadratic monomial

58. Transportation The polynomial $3.675v + 0.096v^2$ is used by transportation officials to estimate the stopping distance in feet for a car whose speed is v miles per hour on flat, dry pavement. What is the stopping distance for a car traveling at 30 miles per hour? **196.65 ft**

Tell whether each statement is sometimes, always, or never true.

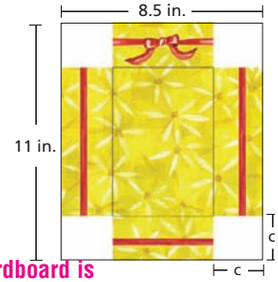
59. A monomial is a polynomial. **always**

60. A trinomial is a 3rd-degree polynomial. **sometimes**

61. A binomial is a trinomial. **never**

62. A polynomial has two or more terms. **sometimes**

63. Geometry A piece of 8.5-by-11-inch cardboard has identical squares cut from its corners. It is then folded into a box with no lid. The volume of the box in cubic inches is $4c^3 - 39c^2 + 93.5c$, where c is the side length of the missing squares in inches.



a. What is the volume of the box if $c = 1$ in.? **58.5 in^3**

b. What is the volume of the box if $c = 1.5$ in.? **66 in^3**

c. What is the volume of the box if $c = 4.25$ in.? **0**

d. Critical Thinking Does your answer to part **c** make sense? Explain why or why not. **Yes; the width of the cardboard is 8.5 in., so 4.25 in. cuts will meet, leaving nothing to fold up.**

Copy and complete the table by evaluating each polynomial for the given values of x.

Polynomial	$x = -2$	$x = 0$	$x = 5$
64. $5x - 6$	$5(-2) - 6 = -16$	$5(0) - 6 = -6$	**19**
65. $x^5 + x^3 + 4x$	**−48**	**0**	**3270**
66. $-10x^2$	**−40**	**0**	**−250**

73. Possible answer: First identify the degree of each term. From left to right, the degrees are 3, 0, 2, 4, and 1. Arrange the terms in order of decreasing degree, and move the plus or minus sign in front of each term with it: $-2x^4 + 4x^3 + 5x^2 - x - 3$.

Give one example of each type of polynomial. **Possible answers given.**

67. quadratic trinomial **$x^2 + 3x - 6$**

68. linear binomial **$5x - 2$**

69. constant monomial **5**

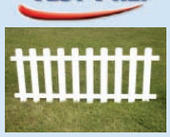
70. cubic monomial **$6x^3$**

71. quintic binomial **$x^5 - 3$**

72. 12th-degree trinomial **$2x^{12} - x + 15$**

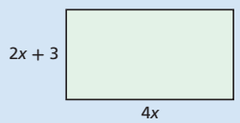
73. Write About It Explain the steps you would follow to write the polynomial $4x^3 - 3 + 5x^2 - 2x^4 - x$ in standard form.

MULTI-STEP TEST PREP



74. a. The perimeter of the rectangle shown is $12x + 6$. What is the degree of this polynomial? **1**

b. The area of the rectangle is $8x^2 + 12x$. What is the degree of this polynomial? **2**



(tl), © Don Johnston/Stone/Getty Images; (bl), HMH

PRACTICE A

PRACTICE C

PRACTICE B

Find the degree and number of terms of each polynomial.

1. $14h^3 + 2h + 10$ — 3, 3
2. $7y - 10y^2$ — 2, 2
3. $2a^2 - 5a + 34 - 6a^4$ — 4, 4

Write each polynomial in standard form. Then, give the leading coefficient.

4. $3x^2 - 2 + 4x^3 - x$ — $4x^3 + 3x^2 - x - 2$, 4
5. $7 - 50j + 3j^3 - 4j^2$ — $3j^3 - 4j^2 - 50j + 7$, 3
6. $6k + 5k^4 - 4k^3 + 3k^2$ — $5k^4 - 4k^3 + 3k^2 + 6k$, 5

Classify each polynomial by its degree and number of terms.

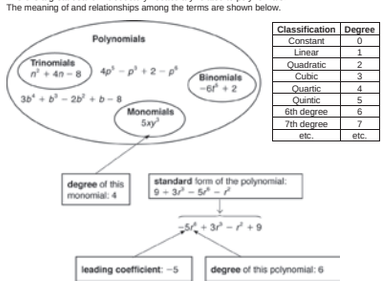
7. $-5t^2 + 10$ — quadratic binomial
8. $8w - 32 + 9w^4$ — quartic trinomial
9. $b - b^3 - 2b^2 + 5b^4$ — quartic polynomial

Evaluate each polynomial for the given value.

10. $3m + 8 - 2m^2$ for $m = -1$ — 7
11. $4y^2 - 6y + 8y^2 - 1$ for $y = -1$ — 9
12. $2w + w^2 - \frac{1}{2}w^2$ for $w = 2$ — 10
13. An egg is thrown off the top of a building. Its height in meters above the ground can be approximated by the polynomial $300 + 2t - 4.9t^2$, where t is the time since it was thrown in seconds.
    a. How high is the egg above the ground after 5 seconds? 187.5 m
    b. How high is the egg above the ground after 6 seconds? 135.6 m

READING STRATEGIES

There is a great deal of introductory vocabulary related to polynomials. The meaning of and relationships among the terms are shown below.



Classification	Degree
Constant	0
Linear	1
Quadratic	2
Cubic	3
Quartic	4
Quintic	5
6th degree	6
7th degree	7
etc.	etc.

Complete each of the following.

1. How many terms are in a trinomial? 3 a monomial? 1
2. Why does $5xy^3$ have a degree of 4? (*Hint:* Look at the exponents on the variables.) The exponents on the variables have a sum of 4.
3. Classify $4n^3 + 6$ by degree. cubic by number of terms. binomial

Use the polynomial $8g + 1 - 4g^2$ to complete the following.

4. Write the polynomial in standard form. $-4g^2 + 8g + 1$
5. What is the leading coefficient? −4
6. What is the degree of the polynomial? 2
7. Classify the polynomial by number of terms. trinomial
8. Classify the polynomial by degree. quadratic

RETEACH

A **monomial** is a number, a variable, or a product of numbers and variables with whole-number exponents. A **polynomial** is a monomial or a sum or difference of monomials.

The degree of the monomial is the sum of the exponents in the monomial.

Find the degree of $8x^2y^3$. $8x^2y^3$ *The exponents are 2 and 3.* The degree of the monomial is $2 + 3 = 5$.

Find the degree of $-4a^5b$. $-4a^5b$ *The exponents are 6 and 1.* The degree of the monomial is $6 + 1 = 7$.

The degree of the polynomial is the degree of the term with the greatest degree.

Find the degree of $2x^4y^2 + 9x^5$. $2x^4y^2 + 9x^5$ — Degree of the polynomial is 7.

Find the degree of $4ab^2 + 9a^3$. $4ab^2 + 9a^3$ — Degree of the polynomial is 3.

The **standard form of a polynomial** is written with the terms in order from the greatest degree to the least degree. The coefficient of the first term is the **leading coefficient.**

Write $5x + 6x^2 + 4 + 2x^4$ in standard form.

$5x + 6x^2 + 4 + 2x^4$ *Find the degree of each term.*

$2x^4 + 6x^2 + 5x + 4$ *Write the terms in order of degree.*

The leading coefficient is 2.

Find the degree of each monomial.

1. $7m^2n^5$ — 8
2. $6xyz$ — 3
3. $4x^2y^2$ — 4

Find the degree of each polynomial.

4. $x^3 + x^3y$ — 6
5. $4x^2y^3 + y^4 + 7$ — 5
6. $x^2 + xy + y$ — 2

Write each polynomial in standard form. Then give the leading coefficient.

7. $x^2 - 5x^4 - 6x^3$ — $-6x^3 - 5x^4 + x^2$, −6
8. $2x + 5x^2 - x^3$ — $-x^3 + 5x^2 + 2x$, −1
9. $8x + 7x^2 - 1$ — $7x^2 + 8x - 1$, 7

75. ///**ERROR ANALYSIS**/// Two students evaluated $4x - 3x^5$ for $x = -2$. Which is incorrect? Explain the error.

75. A is incorrect. The student incorrectly multiplied −3 by −2 before evaluating the power.

A

$4(-2) - 3(-2)^5$
$-8 + 6^5$
$-8 + 7776$
7768

B

$4(-2) - 3(-2)^5$
$-8 - 3(-32)$
$-8 + 96$
88

TEST PREP

76. Which polynomial has the highest degree?

Ⓐ $3x^8 - 2x^7 + x^6$ Ⓑ $5x - 100$ Ⓒ $25x^{10} + 3x^5 - 15$ Ⓓ $134x^2$

77. What is the value of $-3x^3 + 4x^2 - 5x + 7$ when $x = -1$?

Ⓕ 3 Ⓖ 13 Ⓗ 9 Ⓙ 19

78. Short Response A toy rocket is launched from the ground at 75 feet per second. The polynomial $-16t^2 + 75t$ gives the rocket's height in feet after t seconds. Make a table showing the rocket's height after 1 second, 2 seconds, 3 seconds, and 4 seconds. At which of these times will the rocket be the highest?

CHALLENGE AND EXTEND

79. Medicine Doctors and nurses use growth charts and formulas to tell whether a baby is developing normally. The polynomial $0.016m^3 - 0.390m^2 + 4.562m + 50.310$ gives the average length in centimeters of a baby boy between 0 and 10 months of age, where m is the baby's age in months.

79c. The first 3 terms of the polynomial will equal 0, so just look at the constant.

a. What is the average length of a 2-month-old baby boy? a 5-month-old baby boy? Round your answers to the nearest centimeter. **58 cm; 65 cm**

b. What is the average length of a newborn (0-month-old) baby boy? **50.310 cm**

c. How could you find the answer to part **b** without doing any calculations?

80. Consider the binomials $4x^5 + x$, $4x^4 + x$, and $4x^3 + x$.

a. Without calculating, which binomial has the greatest value for $x = 5$? **$4x^5 + x$**

b. Are there any values of x for $4x^3 + x$ which will have the greatest value? Explain.

PROBLEM SOLVING

Write the correct answer.

1. The surface area of a cylinder is given by the polynomial $2\pi r^2 + 2\pi rh$. A cylinder has a radius of 2 centimeters and a height of 5 centimeters. Find the surface area of the cylinder. Use 3.14 for π.
87.92 square centimeters

2. A firework is launched from the ground at a velocity of 180 feet per second. Its height after t seconds is given by the polynomial $-16t^2 + 180t$. Find the height of the firework after 2 seconds and after 5 seconds.
2 s: 296 feet
5 s: 500 feet

3. In the United Kingdom, transportation authorities use the polynomial $\frac{1}{20}v^2 + v$ for calculating the number of feet needed to stop on dry pavement. In the United States, many use the polynomial $0.096v^2$. Both formulas are based on speed v in miles per hour. Calculate the stopping distances for a car traveling 45 miles per hour in both the U.S. and the UK.
UK: 146.25 feet
US: 194.4 feet

4. A piece of cardboard that measures 2 feet by 3 feet can be folded into a box if notches are cut out of the corners. The length of the side of the notch will be the same as the height h of the resulting box. The volume of the box is given by $4h^3 - 10h^2 + 6h$. Find the volume of the box for $h = 0.25$ and $h = 0.5$.
$h = 0.25$: 0.9375 cubic feet
$h = 0.5$: 1 cubic foot

The height of a rocket in meters t seconds after it is launched is approximated by the polynomial $0.5at^2 + vt + h$ where a is always −9.8, v is the initial velocity, and h is the initial height. Use this information with the data in the chart for questions 5 – 7. Select the best answer.

Model Number	Initial Velocity (m/s)
300X	250
Q99	90
4400*i*	125

5. A 300X was launched from a height of 10 meters. What was its height after 3 seconds?
A 715.9 m C 755.5 m
B 745.3 m D 760 m

6. Marie and Bob launched their rockets at the same time from a platform 5 meters above the ground. Marie launched the 4400*i* and Bob launched the Q99. How much higher was Marie's rocket after 2 seconds?
F 35 meters H 140 meters
G 70 meters J 320 meters

7. The 4400*i* was launched from the ground at the same time the Q99 was launched from 175 meters above the ground. After how many seconds were the rockets at the same height?
A 2 s C 5 s
B 4 s D 6 s

CHALLENGE

Match each polynomial with the correct clue. Each polynomial can be used only once. Not every polynomial will be used.

Use these polynomials for 1 – 7.

$x^5 + x^3 + x$	$2x^5y^4 + 3x^2y^5$
$2x^4$	$x^3y - 3x^2y - xy$
$3x + 3y + 3z$	$-3xy^4$
$2xy + 5xz$	$2x^2y + 5xy^2$
$4x^2 - 3x^5 + x$	$4xyz^2 + xyz$

Use these polynomials for 8 – 14.

$4x^2 - 3x^2y$	$x^3 + x^2 + x$
$x^2 + x - 3$	$x^4 + x^3 + x^2$
$-3 + x + 4x^2$	$2x^2 - 3x^5 + 1$
$x^3y - 3xyz + z$	$x - 3$
$-x + 3$	$x^4 + y^4 + z^4 + w^4$

1. I am a monomial with degree 5. Who am I?
$-3xy^4$
2. I am a sum of monomials with degree 8. Who am I?
$2x^4y^4 + 3x^2y^5$
3. I am a trinomial with degree 5. Who am I?
$x^5 + x^3 + x$
4. I am a binomial. Both of my terms have degree 2. Who am I?
$2xy + 5xz$
5. I am a monomial with degree 4. Who am I?
$2x^4$
6. I am a binomial with degree 4. Who am I?
$4xyz^2 + xyz$
7. I am a trinomial. When you put me in standard form, my leading coefficient is −3. Who am I?
$4x^2 - 3x^5 + x$
8. I am a linear expression. My constant is −3. Who am I?
$x - 3$
9. I am a quartic trinomial. I have three different variables. Who am I?
$x^3y - 3xyz + z$
10. I am a cubic binomial. Who am I?
$4x^2 - 3x^2y$
11. I am a quadratic polynomial. I have no constants. Who am I?
$x^2 + y^2 + z^2 + w^2$
12. I am a cubic trinomial with one variable. Who am I?
$x^3 + x^2 + x$
13. I am a quadratic trinomial. When you put me in standard form, my leading coefficient is 4. Who am I?
$-3 + x + 4x^2$
14. I am a quartic trinomial. Who am I?
$x^4 + x^3 + x^2$

TEST PREP DOCTOR In **Exercise 76,** students who chose **A** may have confused degree with number of terms. Students who chose **D** may think that the coefficients are significant when determining degree.

In **Exercise 77,** students who chose **G** probably got −3 for the value of the first term.

Answers

80b. Yes; $0 < x < 1$; raising a number between 0 and 1 to a higher power results in a lesser number. So if x is between 0 and 1, the binomial with the least degree will have the greatest value.

Journal

Have students explain how to classify a polynomial according to its degree and number of terms.

ALTERNATIVE ASSESSMENT

Have students write a linear binomial and a quadratic trinomial, each with only x as a variable. Then have them evaluate each polynomial for $x = 3$.

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6-3 Lesson Quiz

Find the degree of each polynomial.

1. $7a^3b^2 - 2a^4 + 4b - 15$ 5

2. $25x^2 - 3x^4$ 4

Write each polynomial in standard form. Then give the leading coefficient.

3. $24g^3 + 10 + 7g^5 - g^2$
$7g^5 + 24g^3 - g^2 + 10$; 7

4. $14 - x^4 + 3x^2$ $-x^4 + 3x^2 + 14$; −1

Classify each polynomial according to its degree and number of terms.

5. $18x^2 - 12x + 5$
quadratic trinomial

6. $2x^4 - 1$ quartic binomial

7. The polynomial $3.675v + 0.096v^2$ is used to estimate the stopping distance in feet for a car whose speed is v miles per hour on flat, dry pavement. What is the stopping distance for a car traveling at 70 miles per hour?
727.65 ft

Also available online

Organizer

Use with the lesson *Adding and Subtracting Polynomials*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model polynomial addition and subtraction.

Materials: algebra tiles

Online Edition
Algebra Tiles

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

Compare like terms and like tiles. In like terms, the same variables are raised to the same powers, but the coefficients may differ. In like tiles, the size and shape are the same, but the color of the tile (whether it is positive or negative) may differ.

In **Activity 2,** remind students to add the opposite of every term in the second expression.

Encourage students to always arrange their tiles and write their answers in standard form. This will help prevent careless errors.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Learn It Online
State Resources Online

6-4 Algebra LAB

Model Polynomial Addition and Subtraction

You can use algebra tiles to model polynomial addition and subtraction.

Use with Adding and Subtracting Polynomials

KEY

+ = 1
− = −1
+ = x
− = −x
+ = x^2
− = $-x^2$

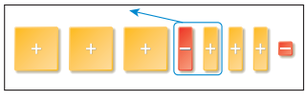
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Activity 1

Use algebra tiles to find $(2x^2 - x) + (x^2 + 3x - 1)$.

MODEL	ALGEBRA
Use tiles to represent all terms from both expressions.	$(2x^2 - x) + (x^2 + 3x - 1)$
Rearrange tiles so that like tiles are together. Like tiles are the same size and shape.	$(2x^2 + x^2) + (-x + 3x) - 1$
 *Remove any zero pairs.*	$3x^2 - x + x + 2x - 1$
The remaining tiles represent the sum.	$3x^2 + 2x - 1$

Try This

Use algebra tiles to find each sum.

1. $(-2x^2 + 1) + (-x^2)$ $-3x^2 + 1$
2. $(3x^2 + 2x + 5) + (x^2 - x - 4)$ $4x^2 + x + 1$
3. $(x - 3) + (2x - 2)$ $3x - 5$
4. $(5x^2 - 3x - 6) + (x^2 + 3x + 6)$ $6x^2$
5. $-5x^2 + (2x^2 + 5x)$ $-3x^2 + 5x$
6. $(x^2 - x - 1) + (6x - 3)$ $x^2 + 5x - 4$

Activity 2

Use algebra tiles to find $(2x^2 + 6) - 4x^2$.

MODEL		ALGEBRA
	Use tiles to represent the terms in the first expression.	$2x^2 + 6$
To subtract $4x^2$, you would remove 4 yellow x^2-tiles, but there are not enough to do this. Remember that subtraction is the same as adding the opposite, so rewrite $(2x^2 + 6) - 4x^2$ as $(2x^2 + 6) + (-4x^2)$.	*Add 4 red x^2-tiles.*	$2x^2 + 6 + (-4x^2)$
	Rearrange tiles so that like tiles are together.	$2x^2 + (-4x^2) + 6$
	Remove zero pairs.	$2x^2 + (-2x^2) + (-2x^2) + 6$
	The remaining tiles represent the difference.	$-2x^2 + 6$

Try This

Use algebra tiles to find each difference.

7. $(6x^2 + 4x) - 3x^2$ **8.** $(2x^2 + x - 7) - 5x$ **9.** $(3x + 6) - 6$

10. $(8x + 5) - (-2x)$ **11.** $(x^2 + 2x) - (-4x^2 + x)$ **12.** $(3x^2 - 4) - (x^2 + 6x)$

13. represents a zero pair. Use algebra tiles to model two other zero pairs.

14. When is it not necessary to "add the opposite" for polynomial subtraction using algebra tiles? **when you have enough tiles to actually remove them to model the subtraction**

Close

Key Concept

Polynomial addition is the same as combining like terms. Polynomial subtraction can be written as polynomial addition by adding the opposite.

Assessment

Journal Have students explain how to use algebra tiles to find $(3x^2 - 4x + 1) - 5x$.

Answers to *Try This*

7. $3x^2 + 4x$

8. $2x^2 - 4x - 7$

9. $3x$

10. $10x + 5$

11. $5x^2 + x$

12. $2x^2 - 6x - 4$

13.

6-4 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Add and subtract polynomials.

Technology Lab
In *Technology Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify each expression by combining like terms.

1. $4x + 2x$ $6x$
2. $3y + 7y$ $10y$
3. $8p - 5p$ $3p$
4. $5n + 6n^2$ not like terms

Simplify each expression.

5. $3(x + 4)$ $3x + 12$
6. $-2(t + 3)$ $-2t - 6$
7. $-1(x^2 - 4x - 6)$ $-x^2 + 4x + 6$

Also available online

Math Humor

Teacher: What is $b + b$?
Shakespeare: Is it $2b$ or not $2b$?

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Learn It Online
State Resources Online

COMMON CORE

6-4 Adding and Subtracting Polynomials

CC.9-12.A.APR.1 Understand that polynomials … are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Objective
Add and subtract polynomials.

Who uses this?
Business owners can add and subtract polynomials that model profit. (See Example 4.)

Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

"This one pretty much sums it up."

EXAMPLE 1 Adding and Subtracting Monomials

Add or subtract.

A $15m^3 + 6m^2 + 2m^3$

$15m^3 + 6m^2 + 2m^3$ *Identify like terms.*
$15m^3 + 2m^3 + 6m^2$ *Rearrange terms so that like terms are together.*
$17m^3 + 6m^2$ *Combine like terms.*

B $3x^2 + 5 - 7x^2 + 12$

$3x^2 + 5 - 7x^2 + 12$ *Identify like terms.*
$3x^2 - 7x^2 + 5 + 12$ *Rearrange terms so that like terms are together.*
$-4x^2 + 17$ *Combine like terms.*

C $0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$

$0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$ *Identify like terms.*
$0.9y^5 - 0.4y^5 + y^5 + 0.5x^5$ *Rearrange terms so that like terms are together.*
$1.5y^5 + 0.5x^5$ *Combine like terms.*

D $2x^2y - x^2y - x^2y$

$2x^2y - x^2y - x^2y$ *All terms are like terms.*
0 *Combine.*

Remember!
Like terms are constants or terms with the same variable(s) raised to the same power(s).

CHECK IT OUT!

Add or subtract.

1a. $2s^2 + 3s^2 + s$
1b. $4z^4 - 8 + 16z^4 + 2$
1c. $2x^8 + 7y^8 - x^8 - y^8$
1d. $9b^3c^2 + 5b^3c^2 - 13b^3c^2$

Polynomials can be added in either vertical or horizontal form.

In vertical form, align the like terms and add:

$$\begin{array}{r} 5x^2 + 4x + 1 \\ +\ 2x^2 + 5x + 2 \\ \hline 7x^2 + 9x + 3 \end{array}$$

In horizontal form, use the Associative and Commutative Properties to regroup and combine like terms:

$$\begin{aligned} &(5x^2 + 4x + 1) + (2x^2 + 5x + 2) \\ &= (5x^2 + 2x^2) + (4x + 5x) + (1 + 2) \\ &= 7x^2 + 9x + 3 \end{aligned}$$

www.cartoonstock.com

1 Introduce

EXPLORATION

Adding and Subtracting Polynomials

An ecologist is studying frogs and toads in a wetlands habitat. She finds that the polynomial $3x^2 + x$ models the frog population and that the polynomial $7x^2 - x$ models the toad population. In both cases, x represents the number of months.

1. Complete the table. For each month, evaluate the two polynomials to find the population of frogs and toads. Then add these values to find the total population.

Month, x	Population of Frogs, $3x^2 + x$	Population of Toads, $7x^2 - x$	Total Population
1	4	6	10
2	14	26	40
3	30	60	90
4	52	108	160
5	80	170	250
6	114	246	360

2. Look for a pattern in the last column. Write a simplified polynomial that gives the total population in month x.

THINK AND DISCUSS

3. **Show** how to find the sum of the polynomials $3x^2 + x$ and $7x^2 - x$.
4. **Describe** how you could find the sum using the terms of the two polynomials that are being added.

Motivate

Display the following items: 4 books, 3 pencils, 2 books, and 5 pencils.

Ask students for a sensible way to group the items. a group of 6 books and a group of 8 pencils

Explain to students that adding and subtracting monomials is done in a similar way.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Adding Polynomials

Add.

A $(2x^2 - x) + (x^2 + 3x - 1)$

$(2x^2 - x) + (x^2 + 3x - 1)$ *Identify like terms.*

$(2x^2 + x^2) + (-x + 3x) + (-1)$ *Group like terms together.*

$3x^2 + 2x - 1$ *Combine like terms.*

B $(-2ab + b) + (2ab + a)$

$(-2ab + b) + (2ab + a)$ *Identify like terms.*

$(-2ab + 2ab) + b + a$ *Group like terms together.*

$0 + b + a$ *Combine like terms.*

$b + a$ *Simplify.*

C $(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$

$(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$ *Identify like terms.*

$(4b^5 + 8b) + (-4b^5 + 7b)$ *Combine like terms in the second polynomial.*

$4b^5 + 8b$
$+ -4b^5 + 7b$ *Use the vertical method.*

$0 + 15b$ *Combine like terms.*

$15b$ *Simplify.*

D $(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$

$(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$ *Identify like terms.*

$20.2y^2 + 6y + 5$
$+ 1.7y^2 + 0y - 8$ *Use the vertical method. Write 0y as a placeholder in the second polynomial.*

$21.9y^2 + 6y - 3$ *Combine like terms.*

Writing Math

When you use the Associative and Commutative Properties to rearrange the terms, the sign in front of each term must stay with that term.

CHECK IT OUT! 2. Add $(5a^3 + 3a^2 - 6a + 12a^2) + (7a^3 - 10a)$. $12a^3 + 15a^2 - 16a$

To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of *each* term in the polynomial:

$$-(2x^3 - 3x + 7) = -2x^3 + 3x - 7$$

EXAMPLE 3 Subtracting Polynomials

Subtract.

A $(2x^2 + 6) - (4x^2)$

$(2x^2 + 6) + (-4x^2)$ *Rewrite subtraction as addition of the opposite.*

$(2x^2 + 6) + (-4x^2)$ *Identify like terms.*

$(2x^2 - 4x^2) + 6$ *Group like terms together.*

$-2x^2 + 6$ *Combine like terms.*

B $(a^4 - 2a) - (3a^4 - 3a + 1)$

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Rewrite subtraction as addition of the opposite.*

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Identify like terms.*

$(a^4 - 3a^4) + (-2a + 3a) - 1$ *Group like terms together.*

$-2a^4 + a - 1$ *Combine like terms.*

COMMON ERROR ALERT

Students often think the terms xy^2 and x^2y are like terms. Write out all factors to show students how they are different: $xy^2 = xyy$, and $x^2y = xxy$.

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Additional Examples

Example 1

Add or subtract.

A. $12p^3 + 11p^2 + 8p^3$ $20p^3 + 11p^2$

B. $5x^2 - 6 - 3x + 8$ $5x^2 - 3x + 2$

C. $t^2 + 2s^2 - 4t^2 - s^2$ $-3t^2 + s^2$

D. $10m^2n + 4m^2n - 8m^2n$ $6m^2n$

Example 2

Add.

A. $(4m^2 + 5) + (m^2 - m + 6)$ $5m^2 - m + 11$

B. $(10xy + x) + (-3xy + y)$ $7xy + x + y$

C. $(6x^2 - 4y) + (3x^2 + 3y - 8x^2 - 2y)$ $x^2 - 3y$

D. $\left(\frac{1}{2}a^2 + b + 2\right) + \left(\frac{3}{2}a^2 - 4b + 5\right)$ $2a^2 - 3b + 7$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you identify like terms?

EXAMPLE 2

- When using a vertical format to add polynomials, what is a placeholder? How is it helpful?

Teaching Tip **Inclusion** Remind students that the Commutative Property of Addition states that you can add numbers in any order. The Associative Property of Addition states that you can group any of the numbers together.

2 Teach

Guided Instruction

Review like terms before beginning this lesson. When rearranging terms, remind students to pay close attention to the sign in front of each term. Point out that in **Example 2C,** like terms in the second polynomial were simplified before the polynomials were added. Students will be less likely to make mistakes when the individual polynomials are simplified first.

Teaching Tip **Visual Cues** Draw different marks around like terms so they stand out.

Reaching All Learners *Through Cooperative Learning*

Prepare a bag with several small slips of paper (at least four times the number of students in class), each with one of the following: x, $2x$, $-3x$, y, $4y$, $-6y$. Have each student randomly select three or four slips of paper. Have students combine, and simplify if possible, their terms and write the resulting polynomial on a piece of paper. Then have students pair up to find the sums and differences of their polynomials. Have students pair up with as many others as time allows.

Teaching Tip **Multiple Representations** Finding the opposite of a polynomial can be thought of as distributing −1.

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Additional Examples

Example 3

Subtract.

A. $(x^3 + 4y) - (2x^3)$ $-x^3 + 4y$

B. $(7m^4 - 2m^2) - (5m^4 - 5m^2 + 8)$
$2m^4 + 3m^2 - 8$

C. $(-10x^2 - 3x + 7) - (x^2 - 9)$
$-11x^2 - 3x + 16$

D. $(9q^2 - 3q) - (q^2 - 5)$
$8q^2 - 3q + 5$

Example 4

A farmer must add the areas of two plots of land to determine the amount of seed to plant. The area of plot A can be represented by $3x^2 + 7x - 5$, and the area of plot B can be represented by $5x^2 - 4x + 11$. Write a polynomial that represents the total area of both plots of land.
$8x^2 + 3x + 6$

Also available online

Mathematical Practices **Questioning Strategies**

EXAMPLE 3

- What is the first step in subtracting polynomials?
- Why are parentheses used when subtracting polynomials?

EXAMPLE 4

- What is the purpose of aligning terms before adding or subtracting polynomials?

Subtract.

C $(3x^2 - 2x + 8) - (x^2 - 4)$

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Rewrite subtraction as addition of the opposite.*

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Identify like terms.*

$3x^2 - 2x + 8$ *Use the vertical method.*

$+ -x^2 + 0x + 4$ *Write 0x as a placeholder.*

$2x^2 - 2x + 12$ *Combine like terms.*

D $(11z^3 - 2z) - (z^3 - 5)$

$(11z^3 - 2z) + (-z^3 + 5)$ *Rewrite subtraction as addition of the opposite.*

$(11z^3 - 2z) + (-z^3 + 5)$ *Identify like terms.*

$11z^3 - 2z + 0$ *Use the vertical method.*

$+ -z^3 + 0z + 5$ *Write 0 and 0z as placeholders.*

$10z^3 - 2z + 5$ *Combine like terms.*

CHECK IT OUT! **3.** Subtract $(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$.

EXAMPLE 4 *Business Application*

The profits of two different manufacturing plants can be modeled as shown, where x is the number of units produced at each plant.

Eastern: $-0.03x^2 + 25x - 1500$

Southern: $-0.02x^2 + 21x - 1700$

Write a polynomial that represents the difference of the profits at the eastern plant and the profits at the southern plant.

$(-0.03x^2 + 25x - 1500)$ *Eastern plant profits*

$-(-0.02x^2 + 21x - 1700)$ *Southern plant profits*

$-0.03x^2 + 25x - 1500$

$+(+0.02x^2 - 21x + 1700)$ *Write subtraction as addition of the opposite.*

$-0.01x^2 + 4x + 200$ *Combine like terms.*

CHECK IT OUT! **4.** Use the information above to write a polynomial that represents the total profits from both plants.

Mathematical Practices

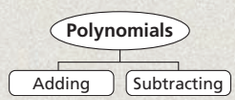
THINK AND DISCUSS

1. Identify the like terms in the following list: $-12x^2$, $-4.7y$, $\frac{1}{5}x^2y$, y, $3xy^2$, $-9x^2$, $5x^2y$, $-12x$

Know it! Note

2. Describe how to find the opposite of $9t^2 - 5t + 8$.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example that shows how to perform the given operation.



3 Close

Summarize

Have students list the steps for adding and subtracting polynomials.

1. Rewrite subtraction as addition if necessary.
2. Identify like terms.
3. Rearrange terms so that like terms are together.
4. Combine like terms.
5. Simplify if necessary.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. $-12x^2$ and $-9x^2$; $-4.7y$ and y; $\frac{1}{5}x^2y$ and $5x^2y$
2. Take the opposite of each term: $-9t^2 + 5t - 8$.
3. See Additional Answers.

6-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1 Add or subtract.

1. $7a^2 - 10a^2 + 9a$ $-3a^2 + 9a$
2. $13x^2 + 9y^2 - 6x^2$ $7x^2 + 9y^2$
3. $0.07r^4 + 0.32r^3 + 0.19r^4$ $0.26r^4 + 0.32r^3$
4. $\frac{1}{4}p^3 + \frac{2}{3}p^3$ $\frac{11}{12}p^3$
5. $5b^3c + b^3c - 3b^3c$ $3b^3c$
6. $-8m + 5 - 16 + 11m$ $3m - 11$

SEE EXAMPLE 2 Add.

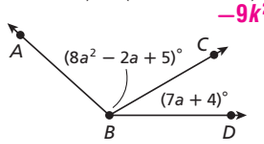
7. $(5n^3 + 3n + 6) + (18n^3 + 9)$
8. $(3.7q^2 - 8q + 3.7) + (4.3q^2 - 2.9q + 1.6)$ $8q^2 - 10.9q + 5.3$
9. $(-3x + 12) + (9x^2 + 2x - 18)$ $9x^2 - x - 6$
10. $(9x^4 + x^3) + (2x^4 + 6x^3 - 8x^4 + x^3)$ $3x^4 + 8x^3$

SEE EXAMPLE 3 Subtract.

11. $(6c^4 + 8c + 6) - (2c^4)$ $4c^4 + 8c + 6$
12. $(16y^2 - 8y + 9) - (6y^2 - 2y + 7y)$ $10y^2 - 13y + 9$
13. $(2r + 5) - (5r - 6)$ $-3r + 11$
14. $(-7k^2 + 3) - (2k^2 + 5k - 1)$ $-9k^2 - 5k + 4$

SEE EXAMPLE 4

15. **Geometry** Write a polynomial that represents the measure of angle ABD. $8a^2 + 5a + 9$



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–24	1
25–28	2
29–32	3
33–34	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Add or subtract.

16. $4k^3 + 6k^2 + 9k^3$
17. $5m + 12n^2 + 6n - 8m$
18. $2.5a^4 - 8.1b^4 - 3.6b^4$
19. $2d^5 + 1 - d^5$
20. $7xy - 4x^2y - 2xy$
21. $-6x^3 + 5x + 2x^3 + 4x^3$
22. $x^2 + x + 3x + 2x^2$
23. $3x^3 - 4 - x^3 - 1$
24. $3b^3 - 2b - 1 - b^3 - b$ $2b^3 - 3b - 1$

Add.

25. $(2t^2 - 8t) + (8t^2 + 9t)$ $10t^2 + t$
26. $(-7x^2 - 2x + 3) + (4x^2 - 9x)$ $-3x^2 - 11x + 3$
27. $(x^5 - x) + (x^4 + x)$ $x^5 + x^4$
28. $(-2z^3 + z + 2z^3 + z) + (3z^3 - 5z^2)$ $3z^3 - 5z^2 + 2z$

Subtract.

29. $(t^3 + 8t^2) - (3t^3)$ $-2t^3 + 8t^2$
30. $(3x^2 - x) - (x^2 + 3x - x)$ $2x^2 - 3x$
31. $(5m + 3) - (6m^3 - 2m^2)$ $-6m^3 + 2m^2 + 5m + 3$
32. $(3s^2 + 4s) - (-10s^2 + 6s)$ $13s^2 - 2s$
33. **Photography** The measurements of a photo and its frame are shown in the diagram. Write a polynomial that represents the width of the photo. $4w^2 + 6w + 4$

34. **Geometry** The length of a rectangle is represented by $4a + 3b$, and its width is represented by $7a - 2b$. Write a polynomial for the perimeter of the rectangle. $22a + 2b$

(frame), Getty Images/PhotoDisc; (photograph), Artville/Getty Images

Answers

7. $23n^3 + 3n + 15$
8. $8q^2 - 10.9q + 5.3$
9. $9x^2 - x - 6$
10. $3x^4 + 8x^3$
16. $13k^3 + 6k^2$
17. $12n^2 + 6n - 3m$
18. $2.5a^4 - 11.7b^4$
19. $d^5 + 1$
20. $-4x^2y + 5xy$
21. $5x$
22. $3x^2 + 4x$
23. $2x^3 - 5$
24. $2b^3 - 3b - 1$

6-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 16–28
Average 16–28, 58
Advanced 16–28, 58

If you finished Examples **1–4**
Basic 16–43, 45–51, 53–56
Average 16–56, 58
Advanced 16–44, 52–62

Homework Quick Check
Quickly check key concepts.
Exercises: 24, 28, 30, 32, 33, 48

Teaching Tip

Auditory For **Exercises 1–6,** suggest to students that they say each term out loud as they do their homework to hear the difference between like and unlike terms.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 15, 33–34, 43, 53, 56–57

Reason abstractly and quantitatively. Exercises 58–62

Construct viable arguments and critique the reasoning of others. Exercises 44–45, 52

Look for and make use of structure. Exercises 46–51

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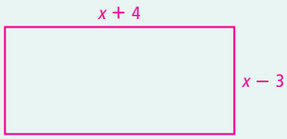
MULTI-STEP TEST PREP **Exercise 53** involves writing expressions for the dimensions of a rectangle. This exercise prepares students for the Multi-Step Test Prep.

Answers

52. No; polynomial addition simply involves combining like terms. No matter what order the terms are combined in, the sum will be the same. Yes; in polynomial subtraction, the subtraction sign is distributed among all terms in the second polynomial, changing all the signs to their opposites.

53a. 

Add or subtract.

35. $(2t - 7) + (-t + 2)$ $t - 5$
36. $(4m^2 + 3m) + (-2m^2)$ $2m^2 + 3m$
37. $(4n - 2) - 2n$ $2n - 2$
38. $(-v - 7) - (-2v)$ $v - 7$
39. $(4x^2 + 3x - 6) + (2x^2 - 4x + 5)$ $6x^2 - x - 1$
40. $(2z^2 - 3z - 3) + (2z^2 - 7z - 1)$ $4z^2 - 10z - 4$
41. $(5u^2 + 3u + 7) - (u^3 + 2u^2 + 1)$ $-u^3 + 3u^2 + 3u + 6$
42. $(-7h^2 - 4h + 7) - (7h^2 - 4h + 11)$ $-14h^2 - 4$

43. **Geometry** The length of a rectangle is represented by $2x + 3$, and its width is represented by $3x + 7$. The perimeter of the rectangle is 35 units. Find the value of x. $\frac{3}{2}$, or 1.5

44. **Write About It** If the parentheses are removed from $(3m^2 - 5m) + (12m^2 + 7m - 10)$, is the new expression equivalent to the original? If the parentheses are removed from $(3m^2 - 5m) - (12m^2 + 7m - 10)$, is the new expression equivalent to the original? Explain.

44. Yes; the simplified form of both expressions is $15m^2 + 2m - 10$. No; the simplified form of the orig. expression is $-9m^2 - 12m + 10$, and the simplified form of the new expression is $-9m^2 + 2m - 10$.

45. **ERROR ANALYSIS** Two students found the sum of the polynomials $(-3n^4 + 6n^3 + 4n^2)$ and $(8n^4 - 3n^2 + 9n)$. Which is incorrect? Explain the error.

A

$$-3n^4 + 6n^3 + 4n^2 + 0n$$
$$+\,8n^4 + 0n^3 - 3n^2 + 9n$$
$$5n^4 + 6n^3 + n^2 + 9n$$

B

$$-3n^4 + 6n^3 + 4n^2$$
$$+\,8n^4 - 3n^2 + 9n$$
$$5n^4 + 3n^3 + 13n^2$$

45. B is incorrect. The student incorrectly tried to combine $6n^3$ and $-3n^2$, which are not like terms, and $4n^2$ and $9n$, which are not like terms.

Copy and complete the table by finding the missing polynomials.

	Polynomial 1	Polynomial 2	Sum
46.	$x^2 - 6$	$3x^2 - 10x + 2$	$4x^2 - 10x - 4$
47.	$12x + 5$	$3x + 6$	$15x + 11$
48.	$x^4 - 3x^2 - 9$	$5x^4 + 8$	$6x^4 - 3x^2 - 1$
49.	$7x^3 - 6x - 3$	$6x + 14$	$7x^3 + 11$
50.	$2x^3 + 5x^2$	$7x^3 - 5x^2 + 1$	$9x^3 + 1$
51.	$2x^2 + x - 5$	$x + x^2 + 6$	$3x^2 + 2x + 1$

52. **Critical Thinking** Does the order in which you add polynomials affect the sum? Does the order in which you subtract polynomials affect the difference? Explain.

MULTI-STEP TEST PREP

53. a. Ian plans to build a fenced dog pen. At first, he planned for the pen to be a square of length x feet on each side, but then he decided that a square may not be best. He added 4 feet to the length and subtracted 3 feet from the width. Draw a diagram to show the dimensions of the new pen.
    b. Write a polynomial that represents the amount of fencing that Ian will need for the new dog pen. $4x + 2$
    c. How much fencing will Ian need if $x = 15$? 62 ft

HMH

PRACTICE A

PRACTICE C

PRACTICE B

Add or subtract.

1. $3m^3 + 8m^2 - 3 + m^3 - 2m^2$ $12m^3 - 2m^2 - 3$
2. $2pg - p^5 - 12pg + 5g - 6p^5$ $-7p^5 - 10pg + 5g$

Add.

3. $3k^2 - 2k + 7$ + $k - 2$ $3k^2 - k + 5$
4. $5x^2 - 2x + 3y$ + $6x^2 + 5x + 6y$ $11x^2 + 3x + 9y$
5. $11hz^3 + 3hz^2 + 8hz$ + $9hz^3 + hz^2 - 3hz$ $20hz^3 + 4hz^2 + 5hz$
6. $(ab^2 + 13b - 4a) + (3ab^2 + a + 7b)$ $4ab^2 + 20b - 3a$
7. $(4x^3 - x^2 + 4x) + (x^3 - x^2 - 4x)$ $5x^3 - 2x^2$

Subtract.

8. $12d^2 + 3dx + x$ − $(-4d^2 + 2dx - 8x)$ $16d^2 + dx + 9x$
9. $2v^5 - 3v^4 - 8$ − $(3v^5 + 2v^4 - 8)$ $-v^5 - 5v^4$
10. $-y^4 + 6ay^2 - y + a$ − $(-6y^4 - 2ay^2 + y)$ $5y^4 + 8ay^2 - 2y + a$
11. $(-r^2 + 8pr - p) - (-12r^2 - 2pr + 8p)$ $11r^2 + 10pr - 9p$
12. $(un - n^2 + 2un^3) - (3un^3 + n^2 + 4un)$ $-3un - 2n^2 - un^3$
13. Antoine is making a banner in the shape of a triangle. He wants to line the banner with a decorative border. How long will the border be? $33b - 8$

14. Darnell and Stephanie have competing refreshment stand businesses. Darnell's profit can be modeled with the polynomial $c^2 + 8c - 100$, where c is the number of items sold. Stephanie's profit can be modeled with the polynomial $2c^2 - 7c - 200$.
    a. Write a polynomial that represents the difference between Stephanie's profit and Darnell's profit. $c^2 - 15c - 100$
    b. Write a polynomial to show how much they can expect to earn if they decided to combine their businesses. $3c^2 + c - 300$

READING STRATEGIES

The process for adding and subtracting polynomials is the same as the process for simplifying linear expressions. Look at the connections below.

Simplify $7 - 3(x + 8) + 4x$. $7 - 3(x + 8) + 4x$

Step 1: Use the Distributive Property.

$= 7 - 3x - 24 + 4x$

Step 2: Rearrange so like terms are together.

$= -3x + 4x + 7 - 24$

Step 3: Combine all the sets of like terms.

$= x - 17$

Subtract $(x^2 + 8x - 4) - (3x^2 - 3x + 2)$. $(x^2 + 8x - 4) - (3x^2 - 3x + 2)$

$= x^2 + 8x - 4 - 3x^2 + 3x - 2$

$= x^2 - 3x^2 + 8x + 3x - 4 - 2$

$= -2x^2 + 11x - 6$

Complete the following based on the examples above.

1. What is being distributed in the linear expression on the left? -3
2. What is being distributed in the polynomial subtraction on the right? -1
3. Identify the sets of like terms that were combined in the expression on the left. $-3x$ and $4x$; 7 and -24
4. Identify the sets of like terms that were combined in the polynomial subtraction on the right. x^2 and $-3x^2$; $8x$ and $3x$; -4 and -2

Add or subtract the polynomials.

5. $5x^3 + 2x + 1 - 3x^3 + 6$ $2x^3 + 2x + 7$
6. $x - 3x^5 + 2x^4 - 5x^5 - x$ $-8x^5 + 2x^4$
7. $(2x^2 + 10x + 4) + (7x^2 + 6x - 2)$ $9x^2 + 16x + 2$
8. $(x^3 - 6) + (9 - 2x^2 + x^3)$ $2x^3 - 2x^2 + 3$
9. $(6x^4 + 8x - 2) - (2x^4 + 6x)$ $4x^4 + 2x - 2$
10. $(3x^2 - 9x) - (x + 2x^2 - 4)$ $-2x^2 + 3x^2 - 10x + 4$

RETEACH

You can add or subtract polynomials by combining like terms.

The following are like terms: $4y$ and $7y$; $8x^2$ and $2x^2$; $7m^3$ and m^3 — same variables raised to same power

The following are not like terms: $3x^2$ and $3x$ — same variable, different exponent; $4y$ and 7 — one with variable, one constant; $8m$ and $3n$ — same power, but different variable

Add $3x^2 + 4x + 5x^2 + 6x$.

$3x^2 + 4x + 5x^2 + 6x$ — Identify like terms.

$3x^2 + 5x^2 + 4x + 6x$ — Rearrange terms so that like terms are together.

$8x^2 + 10x$ — Combine like terms.

Add $(5y^2 + 7y + \frac{2}{3}) + (4y^2 + y + \frac{8}{3})$.

$(5y^2 + 4y^2) + (7y + y) + (\frac{2}{3} + \frac{8}{3})$ — Identify like terms.

Rearrange terms so that like terms are together.

$9y^2 + 8y + 10$ — Combine like terms.

Determine whether the following are like terms. Explain.

1. $4x$ and x^4 — no; same variable raised to different power
2. $5y$ and $7y$ — yes; same variable raised to same power
3. $2z^3$ and $4x^3$ — no; different variable raised to same power

Add.

4. $2y^2 + 3y + 7y + y^2$ $3y^2 + 10y$
5. $8m^4 + 3m - 4m^4$ $4m^4 + 3m$
6. $12x^5 + 10x^4 + 8x^4$ $12x^5 + 18x^4$
7. $(6x^2 + 3x) + (2x^2 + 6x)$ $8x^2 + 9x$
8. $(m^2 - 10m + 5) + (8m + 2)$ $m^2 - 2m + 7$
9. $(6x^3 + 5x) + (4x^3 + x^2 - 2x + 9)$ $10x^3 + x^2 + 3x + 9$
10. $(2y^5 - 6y^3 + 1) + (y^5 + 8y^4 - 2y^3 - 1)$ $3y^5 + 8y^4 - 8y^3$

TEST PREP

54. What is the missing term?

$(-14y^2 + 9y^2 - 12y + 3) + (2y^2 + \square - 6y - 2) = (-3y^2 - 15y + 1)$

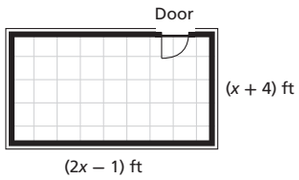
Ⓐ $-6y$ Ⓑ $-3y$ Ⓒ $3y$ Ⓓ $6y$

55. Which is NOT equivalent to $-5t^3 - t$?

Ⓕ $-(5t^3 + t)$ Ⓗ $(t^3 + 6t) - (6t^3 + 7t)$

Ⓖ $(2t^3 - 4t) - (-7t - 3t)$ Ⓙ $(2t^3 - 3t^2 + t) - (7t^3 - 3t^2 + 2t)$

56. Extended Response Tammy plans to put a wallpaper border around the perimeter of her room. She will not put the border across the doorway, which is 3 feet wide.



a. Write a polynomial that represents the number of feet of wallpaper border that Tammy will need. $6x + 3$

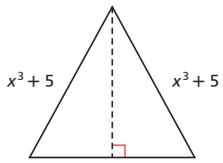
b. A local store has 50 feet of the border that Tammy has chosen. What is the greatest whole-number value of x for which this amount would be enough for Tammy's room? Justify your answer.

c. Determine the dimensions of Tammy's room for the value of x that you found in part **b**. $13 \text{ ft} \times 11 \text{ ft}$

56b. 7; If $x = 7$, Tammy will need $6(7) + 3 = 45$ feet of wallpaper border. However, if $x = 8$, Tammy will need $6(8) + 3 = 51$ feet of wallpaper border, which is more than the store has.

CHALLENGE AND EXTEND

57. Geometry The legs of the isosceles triangle at right measure $(x^3 + 5)$ units. The perimeter of the triangle is $(2x^3 + 3x^2 + 8)$ units. Write a polynomial that represents the measure of the base of the triangle. $3x^2 - 2$



58. Write two polynomials whose sum is $4m^3 + 3m$.

59. Write two polynomials whose difference is $4m^3 + 3m$.

60. Write three polynomials whose sum is $4m^3 + 3m$.

61. Write two monomials whose sum is $4m^3 + 3m$. $4m^3, 3m$

62. Write three trinomials whose sum is $4m^3 + 3m$.
$2m^3 + m^2 + m, m^3 + m^2 + m, m^3 - 2m^2 + m$

58–62. Possible answers given.

58. $2m^3 + 2m$, $2m^3 + m$

59. $5m^3 + 2m$, $m^3 - m$

60. $2m^3 + m$, $m^3 + m$, $m^3 + m$

PROBLEM SOLVING

Write the correct answer.

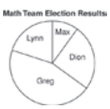
1. There are two boxes in a storage unit. The volume of the first box is $4x^2 + 4x$ cubic units. The volume of the second box is $6x^2 - 18x$ cubic units. Write a polynomial for the total volume of the two boxes.
$10x^3 - 14x^2$ cubic units

2. The recreation field at a middle school is shaped like a rectangle with a length of 15x yards and a width of $10x - 3$ yards. Write a polynomial for the perimeter of the field. Then calculate the perimeter if $x = 2$.
$50x - 6$
94 yards

3. Two cabins on opposite banks of a river are $12x^2 - 7x + 5$ feet apart. One cabin is $9x + 1$ feet from the river. The other cabin is $3x^2 + 4$ feet from the river. Write the polynomial that represents the width of the river where it passes between the two cabins. Then calculate the width if $x = 3$.

$9x^2 - 16x$; 33 feet

The circle graph represents election results for the president of the math team. Use the graph for questions 4–6. Select the best answer.



4. The angle value of Greg's sector can be modeled by $x^2 + 6x + 2$. The angle value of Dion's sector can be modeled by $7x + 20$. Which polynomial represents both sectors combined?
A $x^2 + x + 18$ C $6x^2 + 7x + 18$
B $x^2 + 13x + 22$ D $7x^2 + 6x + 22$

5. The sum of Greg and Lynn's sectors is $2x^2 + 4x - 6$. The sum of Max and Dion's sectors is $10x + 26$. Which polynomial represents how much greater Greg and Lynn's combined sectors are than Max and Dion's?
F $2x^2 + 6x + 32$ H $2x^2 - 6x - 32$
G $2x^2 - 6x + 20$ J $2x^2 + 14x + 20$

6. The sum of Lynn's sector and Max's sector is $2x^2 - 9x - 2$. Max's sector can be modeled by $3x + 6$. Which polynomial represents the angle value of Lynn's sector?
A $2x^2 - 6x + 4$ C $2x^2 - 12x + 8$
B $2x^2 - 6x - 4$ D $2x^2 - 12x - 8$

CHALLENGE

A **polynomial function** is a function whose rule is a polynomial. For example, $P(x) = 2x^2 - 4x^2 + 3x - 2$ is a polynomial function.

Like other functions, polynomial functions can be evaluated by substituting a value for the variable and simplifying.

1. For $P(x) = 2x^2 - 4x^2 + 3x - 2$, find $P(2)$.
4

A variable, or variable expression, can be substituted for the variable in a function as well. For example, if $P(x) = 5x + 3$, then $P(a) = 5a + 3$.

2. For $P(x) = -4x^2 + x - 6$, find $P(b)$.
$-4b^3 + b - 6$

Suppose $P(x) = 3x - 2$. Then $P(c) + P(c + 1) = 3c - 2 + 3(c + 1) - 2$
$= 3c - 2 + 3c + 3 - 2$
$= 6c - 1$

3. For $P(x) = 10 - 4x$, find $P(d) + P(2d) - P(3)$.
$22 - 12d$

4. For $P(x) = -3x^4 - 7x^3 + 2x^2 - x + 8$, find $P(g^2) + P(g)$.
$-3g^8 - 7g^6 - g^4 - 7g^3 + g^2 - g + 16$

Give an example of $Q(x)$ and $S(x)$ for each situation.

5. $Q(x)$ is a quartic binomial. $S(x)$ is a quartic trinomial. $Q(x)$ + $S(x)$ is a quartic trinomial.
$Q(x) = x^4 + 1$; $S(x) = x^4 + x^3 + 1$

6. $Q(x)$ is a cubic trinomial. $S(x)$ is a cubic binomial. $Q(x) - S(x)$ is a quadratic monomial.
$Q(x) = x^3 + x^2 + 1$; $S(x) = x^3 + 1$

7. $Q(x)$ is a cubic trinomial. $S(x)$ is a cubic binomial. $Q(x) + S(x)$ is a quadratic binomial.
$Q(x) = x^3 + x^2 + 1$; $S(x) = -x^3 + x^2$

8. $Q(x)$ is a quintic trinomial. $S(x)$ is a quartic trinomial. $Q(x) + S(x)$ is a quintic polynomial with five terms.
$Q(x) = x^5 + x^4 + 1$; $S(x) = x^4 + x^3 + x^2$

TEST PREP DOCTOR

In **Exercise 54,** after determining that the missing term is a y-term, encourage students to write and solve an algebraic equation to find the coefficient: $-12 + x - 6 = -15$.

Journal

Have students describe two different ways of adding polynomials, using examples in their descriptions.

ALTERNATIVE ASSESSMENT

Have students write four different polynomials, using any of the following as the variable part of the terms: x, y, x^2, y^2. Instruct students to pick any two of the polynomials and find the sum. Then have them find the difference of the remaining two polynomials.

Power Presentations with PowerPoint®

6-4 Lesson Quiz

Add or subtract.

1. $7m^2 + 3m + 4m^2$ $11m^2 + 3m$
2. $(r^2 + s^2) - (5r^2 + 4s^2)$ $-4r^2 - 3s^2$
3. $(10pq + 3p) + (2pq - 5p + 6pq)$ $18pq - 2p$
4. $(14d^2 - 8) + (6d^2 - 2d + 1)$ $20d^2 - 2d - 7$
5. $(2.5ab + 14b) - (-1.5ab + 4b)$ $4ab + 10b$
6. A painter must add the areas of two walls to determine the amount of paint needed. The area of the first wall is modeled by $4x^2 + 12x + 9$, and the area of the second wall is modeled by $36x^2 - 12x + 1$. Write a polynomial that represents the total area of the two walls. $40x^2 + 10$

Also available online

Organizer

Use with the lesson *Multiplying Polynomials*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model polynomial multiplication.

Materials: algebra tiles

Online Edition

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

In **Activity 1,** show students that they would get the same product if they placed the first factor along the top and the second factor along the left side. This is because multiplication is commutative.

In **Activity 3,** remind students that zero pairs are only those whose size and shape are the same, but whose colors are different.

Alternative Approach

Use the transparency mat and transparency algebra tiles (MK).

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

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6-5 Algebra LAB

Model Polynomial Multiplication

Use with Multiplying Polynomials

You can use algebra tiles to multiply polynomials. Use the length and width of a rectangle to represent the factors. The area of the rectangle represents the product.

KEY

$= 1$ $= -1$ $= x$ $= -x$ $= x^2$

REMEMBER

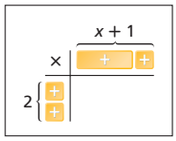
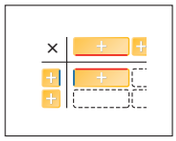
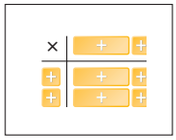
- The product of two values with the same sign is positive.
- The product of two values with different signs is negative.

Activity 1

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Use algebra tiles to find $2(x + 1)$.

MODEL	ALGEBRA
 *Place the first factor in a column along the left side of the grid. This will be the width of the rectangle.* *Place the second factor across the top of the grid. This will be the length of the rectangle.*	$2(x + 1)$
 *Fill in the grid with tiles that have the same width as the tiles in the left column and the same length as the tiles in the top row.*	
 *The area of the rectangle inside the grid represents the product.*	$x + x + 1 + 1$ $2x + 2$

The rectangle has an area of $2x + 2$, so $2(x + 1) = 2x + 2$. Notice that this is the same product you would get by using the Distributive Property to multiply $2(x + 1)$.

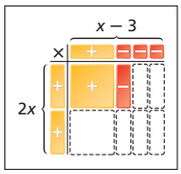
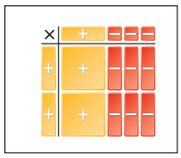
Try This

Use algebra tiles to find each product.

1. $3(x + 2)$ $3x + 6$
2. $2(2x + 1)$ $4x + 2$
3. $3(x + 1)$ $3x + 3$
4. $3(2x + 2)$ $6x + 6$

Activity 2

Use algebra tiles to find $2x(x-3)$.

MODEL		ALGEBRA
	*Place tiles to form the length and width of a rectangle and fill in the rectangle.* *The product of two values with the same sign (same color) is positive (yellow).* *The product of two values with different signs (different colors) is negative (red).*	$2x(x-3)$
	*The area of the rectangle inside the grid represents the product.* *The rectangle has an area of $2x^2 - 6x$, so $2x(x-3) = 2x^2 - 6x$.*	$x^2 + x^2 - x - x - x - x - x - x$ $2x^2 - 6x$

Try This

Use algebra tiles to find each product.

5. $3x(x-2)$
$3x^2 - 6x$

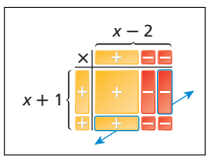
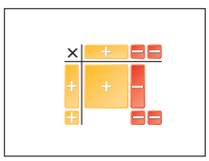
6. $x(2x-1)$
$2x^2 - x$

7. $x(x+1)$
$x^2 + x$

8. $(8x+5)(-2x)$
$-16x^2 - 10x$

Activity 3

Use algebra tiles to find $(x+1)(x-2)$.

MODEL		ALGEBRA
	*Place tiles for each factor to form the length and width of a rectangle.* *Fill in the grid and remove any zero pairs.*	$(x+1)(x-2)$ $x^2 - x - x + x - 1 - 1$
	*The area inside the grid represents the product.* *The remaining area is $x^2 - x - 2$, so $(x+1)(x-2) = x^2 - x - 2$.*	$x^2 - x - 1 - 1$ $x^2 - x - 2$

Try This

Use algebra tiles to find each product.

9. $(x+2)(x-3)$
$x^2 - x - 6$

10. $(x-1)(x+3)$
$x^2 + 2x - 3$

11. $(x-2)(x-3)$
$x^2 - 5x + 6$

12. $(x+1)(x+2)$
$x^2 + 3x + 2$

Close

Key Concept

When multiplication is being modeled with algebra tiles, the dimensions of a rectangle represent the factors, and the area of the rectangle represents the product. The area of the rectangle can sometimes be simplified by removing zero pairs.

Assessment

Journal When using algebra tiles to model polynomial multiplication, have students explain how to determine which tiles should be placed inside the multiplication grid to form the rectangle (product). Explanations should include how to determine sizes, shapes, and colors of tiles.

6-5 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Multiply polynomials.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. 3^2 9
2. 2^4 16
3. 10^2 100
4. $2^3 \cdot 2^4$ 2^7
5. $y^5 \cdot y^4$ y^9
6. $(5^3)^2$ 5^6
7. $(x^2)^4$ x^8
8. $-4(x - 7)$ $-4x + 28$

Also available online

Math Humor

Student A: What is *u* times *r* times *r*?

Student B: ur^2.

Student A: No, I'm not!

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Learn It Online
State Resources Online

COMMON CORE

6-5 Multiplying Polynomials

CC.9-12.A.APR.1 Understand that polynomials … are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Objective
Multiply polynomials.

Why learn this?
You can multiply polynomials to write expressions for areas, such as the area of a dulcimer. (See Example 5.)

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

EXAMPLE 1 Multiplying Monomials

Multiply.

A $(5x^2)(4x^3)$

$(5x^2)(4x^3)$

$(5 \cdot 4)(x^2 \cdot x^3)$ *Group factors with like bases together.*

$20x^5$ *Multiply.*

B $(-3x^3y^2)(4xy^5)$

$(-3x^3y^2)(4xy^5)$

$(-3 \cdot 4)(x^3 \cdot x)(y^2 \cdot y^5)$ *Group factors with like bases together.*

$-12x^4y^7$ *Multiply.*

C $\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2} \cdot 6\right)(a^3 \cdot a^2)(b \cdot b^2)(c^2)$ *Group factors with like bases together.*

$3a^5b^3c^2$ *Multiply.*

Remember!
When multiplying powers with the same base, keep the base and add the exponents.
$x^2 \cdot x^3 = x^{2+3} = x^5$

CHECK IT OUT! **Multiply.**

1a. $(3x^3)(6x^2)$ $18x^5$

1b. $(2r^2t)(5t^3)$ $10r^2t^4$

1c. $\left(\frac{1}{3}x^2y\right)(12x^3z^2)(y^4z^5)$ $4x^5y^5z^7$

To multiply a polynomial by a monomial, use the Distributive Property.

EXAMPLE 2 Multiplying a Polynomial by a Monomial

Multiply.

A $5(2x^2 + x + 4)$

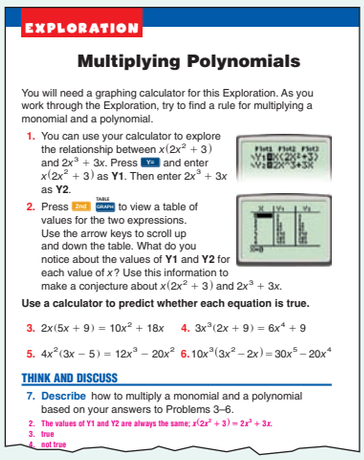
$5(2x^2 + x + 4)$

$(5)2x^2 + (5)x + (5)4$ *Distribute 5.*

$10x^2 + 5x + 20$ *Multiply.*

© Pat and Chuck Blackley

1 Introduce



EXPLORATION

Multiplying Polynomials

You will need a graphing calculator for this Exploration. As you work through the Exploration, try to find a rule for multiplying a monomial and a polynomial.

1. You can use your calculator to explore the relationship between $x(2x^2 + 3)$ and $2x^3 + 3x$. Press Y= and enter $x(2x^2 + 3)$ as **Y1**. Then enter $2x^3 + 3x$ as **Y2**.
2. Press 2nd GRAPH (TABLE) to view a table of values for the two expressions. Use the arrow keys to scroll up and down the table. What do you notice about the values of **Y1** and **Y2** for each value of *x*? Use this information to make a conjecture about $x(2x^2 + 3)$ and $2x^3 + 3x$.

Use a calculator to predict whether each equation is true.

3. $2x(5x + 9) = 10x^2 + 18x$
4. $3x^3(2x + 9) = 6x^4 + 9$
5. $4x^2(3x - 5) = 12x^3 - 20x^2$
6. $10x^3(3x^2 - 2x) = 30x^5 - 20x^4$

THINK AND DISCUSS

7. **Describe** how to multiply a monomial and a polynomial based on your answers to Problems 3–6.

2. The values of Y1 and Y2 are always the same; $x(2x^2 + 3) = 2x^3 + 3x$.
3. true
4. not true

Motivate

Ask students to explain how to find the area of a rectangle with length 5 cm and width 7 cm.
$A = \ell w$, so $A = (5)(7) = 35$ cm^2

Have students explain how to find the area of a rectangle with length x and width 10.
$A = \ell w$, so $A = (x)(10) = 10x$

Explain to students that in this lesson they will learn to describe area when the dimensions are polynomials.

Explorations and answers are provided in your online resources.

Multiply.

B $2x^2y(3x - y)$

$(2x^2y)(3x - y)$

$(2x^2y)3x + (2x^2y)(-y)$ *Distribute $2x^2y$.*

$(2 \cdot 3)(x^2 \cdot x)y + 2(-1)(x^2)(y \cdot y)$ *Group like bases together.*

$6x^3y - 2x^2y^2$ *Multiply.*

C $4a(a^2b + 2b^2)$

$4a(a^2b + 2b^2)$

$(4a)a^2b + (4a)2b^2$ *Distribute 4a.*

$(4)(a \cdot a^2)(b) + (4 \cdot 2)(a)(b^2)$ *Group like bases together.*

$4a^3b + 8ab^2$ *Multiply.*

CHECK IT OUT! **Multiply.**

2a. $2(4x^2 + x + 3)$ $8x^2 + 2x + 6$

2b. $3ab(5a^2 + b)$ $15a^3b + 3ab^2$

2c. $5r^2s^2(r - 3s)$ $5r^3s^2 - 15r^2s^3$

To multiply a binomial by a binomial, you can apply the Distributive Property more than once:

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ *Distribute.*

$= x(x + 2) + 3(x + 2)$

$= x(x) + x(2) + 3(x) + 3(2)$ *Distribute again.*

$= x^2 + 2x + 3x + 6$ *Multiply.*

$= x^2 + 5x + 6$ *Combine like terms.*

Another method for multiplying binomials is called the FOIL method.

1. Multiply the **First** terms. $(x+3)(x+2) \rightarrow x \cdot x = x^2$
2. Multiply the **Outer** terms. $(x+3)(x+2) \rightarrow x \cdot 2 = 2x$
3. Multiply the **Inner** terms. $(x+3)(x+2) \rightarrow 3 \cdot x = 3x$
4. Multiply the **Last** terms. $(x+3)(x+2) \rightarrow 3 \cdot 2 = 6$

$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

F O I L

2 Teach

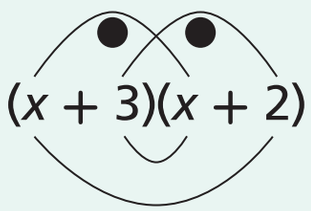
Guided Instruction

Review the multiplication properties of exponents before beginning this lesson. Encourage students to write each step carefully to avoid making mistakes when distributing. Arrows can be drawn to help keep track of the terms that have been multiplied. Remind students that all of the methods presented in this lesson will give correct answers. Students should choose the method they are most comfortable with.

Reaching All Learners

Through Visual Cues

Show students the "FOIL face" to help them keep track of which terms to multiply.



COMMON ERROR ALERT

When a binomial is raised to a power, many students make the following mistake: $(x + 4)^2 = x^2 + 4^2$. Encourage students to write the expression as the product of two binomials first and then multiply. For example, $(x + 4)^2 = (x + 4)(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16$.

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Additional Examples

Example 1

Multiply.

A. $(6y^3)(3y^5)$ $18y^8$

B. $(3mn^2)(9m^2n)$ $27m^3n^3$

C. $\left(\frac{1}{4}s^2t^2\right)(st)(-12st^2)$ $-3s^4t^5$

Example 2

Multiply.

A. $4(3x^2 + 4x - 8)$ $12x^2 + 16x - 32$

B. $6pq(2p - q)$ $12p^2q - 6pq^2$

C. $\frac{1}{2}x^2y(6xy + 8x^2y^2)$ $3x^3y^2 + 4x^4y^3$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How is multiplying monomials different from adding monomials?
- When multiplying monomials with exponents, why do you add the exponents?

EXAMPLE 2

- Will a monomial times a binomial always be a binomial?
- Will a monomial times a trinomial always be a trinomial? Explain.

Teaching Tip **Inclusion** When using the FOIL method, encourage students to write the product after drawing each arrow rather than drawing all of the arrows at once. State: "Draw the F arrow and write the product, draw the O arrow and write the product," and so on.

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Additional Examples

Example 3

Multiply.

A. $(s + 4)(s - 2)$ $s^2 + 2s - 8$

B. $(x - 4)^2$ $x^2 - 8x + 16$

C. $(8m^2 - n)(m^2 - 3n)$
$8m^4 - 25m^2n + 3n^2$

Also available online

Questioning Strategies

EXAMPLE 3

- What terms from FOIL can often be combined?
- What do the signs from the last terms in each binomial tell you about the signs in the answer?

Math Background In **Example 3C,** remind students that the terms $6a^2b$ and $-ba^2$ are like terms due to the Commutative Property of Multiplication.

EXAMPLE 3 Multiplying Binomials

Multiply.

A $(x + 2)(x - 5)$

$(x + 2)(x - 5)$

$x(x - 5) + 2(x - 5)$ *Distribute.*

$x(x) + x(-5) + 2(x) + 2(-5)$ *Distribute again.*

$x^2 - 5x + 2x - 10$ *Multiply.*

$x^2 - 3x - 10$ *Combine like terms.*

Helpful Hint

In the expression $(x + 5)^2$, the base is $(x + 5)$.
$(x + 5)^2 = (x + 5)(x + 5)$

B $(x + 5)^2$

$(x + 5)(x + 5)$ *Write as a product of two binomials.*

$(x \cdot x) + (x \cdot 5) + (5 \cdot x) + (5 \cdot 5)$ *Use the FOIL method.*

$x^2 + 5x + 5x + 25$ *Multiply.*

$x^2 + 10x + 25$ *Combine like terms.*

C $(3a^2 - b)(a^2 - 2b)$

$3a^2(a^2) + 3a^2(-2b) - b(a^2) - b(-2b)$ *Use the FOIL method.*

$3a^4 - 6a^2b - a^2b + 2b^2$ *Multiply.*

$3a^4 - 7a^2b + 2b^2$ *Combine like terms.*

CHECK IT OUT! **3a.** $(a + 3)(a - 4)$ $a^2 - a - 12$ **3b.** $(x - 3)^2$ $x^2 - 6x + 9$ **3c.** $(2a - b^2)(a + 4b^2)$ $2a^2 + 7ab^2 - 4b^4$

To multiply polynomials with more than two terms, you can use the Distributive Property several times. Multiply $(5x + 3)$ by $(2x^2 + 10x - 6)$:

$$(5x + 3)(2x^2 + 10x - 6) = 5x(2x^2 + 10x - 6) + 3(2x^2 + 10x - 6)$$
$$= 5x(2x^2 + 10x - 6) + 3(2x^2 + 10x - 6)$$
$$= 5x(2x^2) + 5x(10x) + 5x(-6) + 3(2x^2) + 3(10x) + 3(-6)$$
$$= 10x^3 + 50x^2 - 30x + 6x^2 + 30x - 18$$
$$= 10x^3 + 56x^2 - 18$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length $(2x^2 + 10x - 6)$ and width $(5x + 3)$:

	$2x^2$	$+ 10x$	$- 6$
$5x$	$10x^3$	$50x^2$	$-30x$
$+ 3$	$6x^2$	$30x$	-18

Write the product of the monomials in each row and column.

To find the product, add all of the terms inside the rectangle by combining like terms and simplifying if necessary.

$$10x^3 + 6x^2 + 50x^2 + 30x - 30x - 18$$
$$10x^3 + 56x^2 - 18$$

Reaching All Learners Through Modeling

A rectangle model can also be used to display the FOIL method.

	x	$+ 3$
x	x^2	$3x$
$+ 2$	$2x$	6

This model shows that $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$.

This rectangle model is useful because it can also be used for factoring polynomials.

Another method that can be used to multiply polynomials with more than two terms is the vertical method. This is similar to methods used to multiply whole numbers.

$2x^2 + 10x - 6$

$\times \quad 5x + 3$

$6x^2 + 30x - 18$ *Multiply each term in the top polynomial by 3.*

$+ 10x^3 + 50x^2 - 30x$ *Multiply each term in the top polynomial by 5x, and align like terms.*

$10x^3 + 56x^2 + 0x - 18$ *Combine like terms by adding vertically.*

$10x^3 + 56x^2 - 18$ *Simplify.*

EXAMPLE 4 Multiplying Polynomials

Multiply.

Helpful Hint

A polynomial with *m* terms multiplied by a polynomial with *n* terms has a product that, before simplifying, has *mn* terms. In Example 4A, there are 2 • 3, or 6, terms before simplifying.

A $(x+2)(x^2-5x+4)$

$(x+2)(x^2-5x+4)$

$x(x^2-5x+4) + 2(x^2-5x+4)$ *Distribute.*

$x(x^2) + x(-5x) + x(4) + 2(x^2) + 2(-5x) + 2(4)$ *Distribute again.*

$x^3 + 2x^2 - 5x^2 - 10x + 4x + 8$ *Simplify.*

$x^3 - 3x^2 - 6x + 8$ *Combine like terms.*

B $(3x-4)(-2x^3+5x-6)$

$(3x-4)(-2x^3+5x-6)$

$-2x^3 + 0x^2 + 5x - 6$ *Add $0x^2$ as a placeholder.*

$\times \quad 3x - 4$

$8x^3 + 0x^2 - 20x + 24$ *Multiply each term in the top polynomial by −4.*

$+ -6x^4 + 0x^3 + 15x^2 - 18x$ *Multiply each term in the top polynomial by 3x, and align like terms.*

$-6x^4 + 8x^3 + 15x^2 - 38x + 24$ *Combine like terms by adding vertically.*

C $(x-2)^3$

$[(x-2)(x-2)](x-2)$ *Write as the product of three binomials.*

$[x \cdot x + x(-2) + (-2)x + (-2)(-2)](x-2)$ *Use the FOIL method on the first two factors.*

$(x^2 - 2x - 2x + 4)(x-2)$ *Multiply.*

$(x^2 - 4x + 4)(x-2)$ *Combine like terms.*

$(x-2)(x^2-4x+4)$ *Use the Commutative Property of Multiplication.*

$x(x^2-4x+4) + (-2)(x^2-4x+4)$ *Distribute.*

$x(x^2) + x(-4x) + x(4) + (-2)(x^2) + (-2)(-4x) + (-2)(4)$ *Distribute again.*

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$ *Simplify.*

$x^3 - 6x^2 + 12x - 8$ *Combine like terms.*

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Additional Examples

Example 4

Multiply.

A. $(x-5)(x^2+4x-6)$
$x^3 - x^2 - 26x + 30$

B. $(2x-5)(-4x^2-10x+3)$
$-8x^3 + 56x - 15$

C. $(x+3)^3$ $x^3 + 9x^2 + 27x + 27$

D. $(3x+1)(x^3+4x^2-7)$ $3x^4 + 13x^3 + 4x^2 - 21x - 7$

Also available online

Questioning Strategies

EXAMPLE 4

- How is multiplying a polynomial by a binomial similar to multiplying a binomial by a binomial? How is it different?

Teaching Tip **Critical Thinking** Ask students to find similarities between polynomials and real numbers. Discuss the types of operations you can use with each.

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Additional Examples

Example 5

The width of a rectangular prism is 3 feet less than the height, and the length of the prism is 4 feet more than the height.

a. Write a polynomial that represents the area of the base of the prism.
$h^2 + h - 12$

b. Find the area of the base when the height is 5 feet.
18 ft²

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 5

- What types of polynomials did you multiply?

Multiply.

D $(2x + 3)(x^2 - 6x + 5)$

	x^2	$-6x$	$+5$
$2x$	$2x^3$	$-12x^2$	$10x$
$+3$	$3x^2$	$-18x$	15

Write the product of the monomials in each row and column.

$2x^3 + 3x^2 - 12x^2 - 18x + 10x + 15$ *Add all terms inside the rectangle.*

$2x^3 - 9x^2 - 8x + 15$ *Combine like terms.*

CHECK IT OUT!

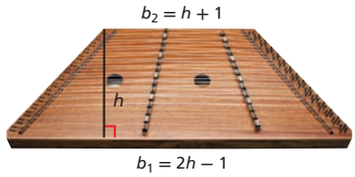
Multiply.

4a. $(x + 3)(x^2 - 4x + 6)$ $x^3 - x^2 - 6x + 18$

4b. $(3x + 2)(x^2 - 2x + 5)$ $3x^3 - 4x^2 + 11x + 10$

EXAMPLE 5 *Music Application*

A dulcimer is a musical instrument that is sometimes shaped like a trapezoid.



A Write a polynomial that represents the area of the dulcimer shown.

$A = \frac{1}{2}h(b_1 + b_2)$ *Write the formula for area of a trapezoid.*

$= \frac{1}{2}h[(2h - 1) + (h + 1)]$ *Substitute $2h - 1$ for b_1 and $h + 1$ for b_2.*

$= \frac{1}{2}h(3h)$ *Combine like terms.*

$= \frac{3}{2}h^2$ *Simplify.*

The area is represented by $\frac{3}{2}h^2$.

B Find the area of the dulcimer when the height is 22 inches.

$A = \frac{3}{2}h^2$ *Use the polynomial from part a.*

$= \frac{3}{2}(22)^2$ *Substitute 22 for h.*

$= \frac{3}{2}(484) = 726$

The area is 726 square inches.

CHECK IT OUT!

5. The length of a rectangle is 4 meters shorter than its width.

a. Write a polynomial that represents the area of the rectangle.

b. Find the area of the rectangle when the width is 6 meters.

5a. $x^2 - 4x$

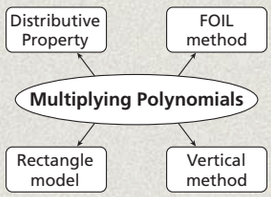
5b. 12 m²

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Compare the vertical method for multiplying polynomials with the vertical method for multiplying whole numbers.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, multiply two polynomials using the given method.



3 Close

Summarize

Ask students to multiply the following using any of the methods learned in this lesson.

$(5r^2s)(9rs)$ $45r^3s^2$

$3x^2(x^3 - 10)$ $3x^5 - 30x^2$

$(y + 2)(y - 5)$ $y^2 - 3y - 10$

$(x + 5y)(xy + 4x + 7y)$

$x^2y + 4x^2 + 27xy + 5xy^2 + 35y^2$

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: Both numbers and polynomials are set up in 2 rows and require you to multiply each item in the top row by an item in the bottom row. In the end, you add vertically to get the answer. When you are multiplying polynomials, the items are monomial terms. When you are multiplying numbers, the items are digits.
2. See Additional Answers.

6-5 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Multiply.

SEE EXAMPLE 1

1. $(2x^2)(7x^4)$
2. $(-5mn^3)(4m^2n^2)$
3. $(6rs^2)(s^3t^2)\left(\frac{1}{2}r^4t^3\right)$
4. $\left(\frac{1}{3}a^5\right)(12a)$
5. $(-3x^4y^2)(-7x^3y)$
6. $(-2pq^3)(5p^2q^2)(-3q^4)$

SEE EXAMPLE 2

7. $4(x^2+2x+1)$
8. $3ab(2a^2+3b^3)$
9. $2a^3b(3a^2b+ab^2)$
10. $-3x(x^2-4x+6)$
11. $5x^2y(2xy^3-y)$
12. $5m^2n^3 \cdot mn^2(4m-n)$

SEE EXAMPLE 3

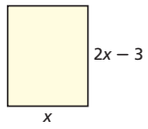
13. $(x+1)(x-2)$
14. $(x+1)^2$
15. $(x-2)^2$
16. $(y-3)(y-5)$
17. $(4a^3-2b)(a-3b^2)$
18. $(m^2-2mn)(3mn+n^2)$

SEE EXAMPLE 4

19. $(x+5)(x^2-2x+3)$
20. $(3x+4)(x^2-5x+2)$
21. $(2x-4)(-3x^3+2x-5)$
22. $(-4x+6)(2x^3-x^2+1)$
23. $(x-5)(x^2+x+1)$
24. $(a+b)(a-b)(b-a)$

SEE EXAMPLE 5

25. **Photography** The length of a rectangular photograph is 3 inches less than twice the width. $2x^2 - 3x$



   a. Write a polynomial that represents the area of the photograph.
   b. Find the area of the photograph when the width is 4 inches. 20 in^2

PRACTICE AND PROBLEM SOLVING

Independent Practice

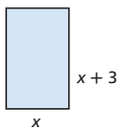
For Exercises	See Example
26–34	1
35–43	2
44–52	3
53–61	4
62	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Multiply.

26. $(3x^2)(8x^5)$
27. $(-2r^3s^4)(6r^2s)$
28. $(15xy^2)\left(\frac{1}{3}x^2z^3\right)(y^3z^4)$
29. $(-2a^3)(-5a)$
30. $(6x^3y^2)(-2x^2y)$
31. $(-3a^2b)(-2b^3)(-a^3b^2)$
32. $(7x^2)(xy^5)(2x^3y^2)$
33. $(-4a^3bc^2)(a^3b^2c)(3ab^4c^5)$
34. $(12mn^2)(2m^2n)(mn)$
35. $9s(s+6)$
36. $9(2x^2-5x)$
37. $3x(9x^2-4x)$
38. $3(2x^2+5x+4)$
39. $5s^2t^3(2s-3t^2)$
40. $x^2y^3 \cdot 5x^2y(6x+y^2)$
41. $-5x(2x^2-3x-1)$
42. $-2a^2b^3(3ab^2-a^2b)$
43. $-7x^3y \cdot x^2y^2(2x-y)$
44. $(x+5)(x-3)$
45. $(x+4)^2$
46. $(m-5)^2$
47. $(5x-2)(x+3)$
48. $(3x-4)^2$
49. $(5x+2)(2x-1)$
50. $(x-1)(x-2)$
51. $(x-8)(7x+4)$
52. $(2x+7)(3x+7)$
53. $(x+2)(x^2-3x+5)$
54. $(2x+5)(x^2-4x+3)$
55. $(5x-1)(-2x^3+4x-3)$
56. $(x-3)(x^2-5x+6)$
57. $(2x^2-3)(4x^3-x^2+7)$
58. $(x-4)^3$
59. $(x-2)(x^2+2x+1)$
60. $(2x+10)(4-x+6x^3)$
61. $(1-x)^3$

62. **Geometry** The length of the rectangle at right is 3 feet longer than its width. $x^2 + 3x$



   a. Write a polynomial that represents the area of the rectangle.
   b. Find the area of the rectangle when the width is 5 feet. 40 ft^2

63. A square tabletop has side lengths of $(4x-6)$ units. Write a polynomial that represents the area of the tabletop. $16x^2 - 48x + 36$

15. $x^2 - 4x + 4$
16. $y^2 - 8y + 15$
17. $4a^4 - 2ab - 12a^3b^2 + 6b^3$
18. $3m^3n - 5m^2n^2 - 2mn^3$
19. $x^3 + 3x^2 - 7x + 15$
20. $3x^3 - 11x^2 - 14x + 8$
21. $-6x^4 + 12x^3 + 4x^2 - 18x + 20$
22. $-8x^4 + 16x^3 - 6x^2 - 4x + 6$
23. $x^3 - 4x^2 - 4x - 5$
24. $-a^3 + a^2b + ab^2 - b^3$
26. $24x^7$
27. $-12r^5s^5$
28. $5x^3y^5z^7$
29. $10a^4$
30. $-12x^5y^3$
31. $-6a^5b^6$
32. $14x^6y^7$
33. $-12a^7b^7c^8$
34. $24m^4n^4$
35. $9s^2 + 54s$
36. $18x^2 - 45x$
37. $27x^3 - 12x^2$
38. $6x^2 + 15x + 12$
39. $10s^3t^3 - 15s^2t^5$
40. $30x^5y^4 + 5x^4y^6$
41. $-10x^3 + 15x^2 + 5x$
42. $-6a^3b^5 + 2a^4b^4$
43. $-14x^6y^3 + 7x^5y^4$
44. $x^2 + 2x - 15$
45. $x^2 + 8x + 16$
46. $m^2 - 10m + 25$
47. $5x^2 + 13x - 6$
48. $9x^2 - 24x + 16$
49–61. See Additional Answers.

6-5 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
Basic 26–52, 70–78
Average 26–52, 70–78
Advanced 26–52, 70–78, 90

If you finished Examples 1–5
Basic 26–69, 75–84, 87–89
Average 26–65, 70–80 even, 82–93
Advanced 26–64, 70–80 even, 82–97

Homework Quick Check
Quickly check key concepts.
Exercises: 30, 42, 46, 52, 56, 62, 69–97

Answers

1. $14x^6$
2. $-20m^3n^5$
3. $3r^5s^5t^5$
4. $4a^6$
5. $21x^7y^3$
6. $30p^3q^9$
7. $4x^2 + 8x + 4$
8. $6a^3b + 9ab^4$
9. $6a^5b^2 + 2a^4b^3$
10. $-3x^3 + 12x^2 - 18x$
11. $10x^3y^4 - 5x^2y^2$
12. $20m^4n^5 - 5m^3n^6$
13. $x^2 - x - 2$
14. $x^2 + 2x + 1$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 25, 62–64, 69, 82, 84, 94

Reason abstractly and quantitatively. Exercises 66–68, 93

Construct viable arguments and critique the reasoning of others. Exercises 83, 85

Attend to precision. Exercise 86

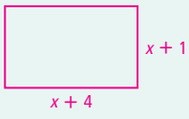
Look for and make use of structure. Exercises 65, 96–97

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 64** involves multiplying polynomials. This exercise prepares students for the Multi-Step Test Prep.

Answers

64a.



65b. $x^4 - x^3 + 2x^2 - 2x$

c. $x^4 - 5x^3 + 6x^2 + x - 3$

70. $6a^9$

71. $2x^2 - 7x - 30$

72. $3g^2 + 14g - 5$

73. $8x^2 - 16xy + 6y^2$

74. $x^2 - 9$

75. $6x^2 - 9x - 6$

76. $x^2 - 6x - 40$

77. $x^3 + 3x^2$

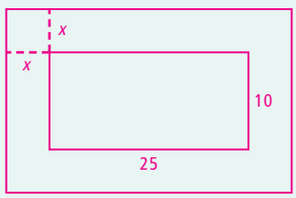
78. $x^3 + 3x^2 + 2x$

79. $2x^3 - 7x^2 - 10x + 24$

80. $a^3 - a^2b - ab^2 + b^3$

81. $8p^3 - 36p^2q + 54pq^2 - 27q^3$

82a.



83. Possible answer: Each letter in FOIL represents a pair of terms in a certain position within the factors. The letters must account for every pairing of terms while describing first, outside, inside, and last positions. This is only possible with 2 binomials.

MULTI-STEP TEST PREP

64. **a.** Marie is creating a garden. She designs a rectangular garden with a length of $(x + 4)$ feet and a width of $(x + 1)$ feet. Draw a diagram of Marie's garden with the length and width labeled.

b. Write a polynomial that represents the area of Marie's garden. $x^2 + 5x + 4$

c. What is the area when $x = 4$? 40 ft²

65. Copy and complete the table below.

	A	Degree of A	B	Degree of B	A • B	Degree of A • B
	$2x^2$	2	$3x^5$	5	$6x^7$	7
a.	$5x^3$	3	$2x^2 + 1$	2	$10x^5 + 5x^3$	5
b.	$x^2 + 2$	2	$x^2 - x$	2		4
c.	$x - 3$	1	$x^3 - 2x^2 + 1$	3		4

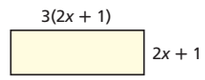
d. Use the results from the table to complete the following: The product of a polynomial of degree m and a polynomial of degree n has a degree of ____. $m + n$

Geometry **Write a polynomial that represents the area of each rectangle.**

66.

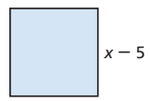
$8x^2 + 12x$

67.



$12x^2 + 12x + 3$

68.



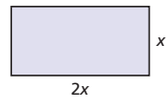
$x^2 - 10x + 25$

LINK

Sports

Team handball is a game with elements of soccer and basketball. It originated in Europe in the 1900s and was first played at the Olympics in 1936 with teams of 11 players. Today, a handball team consists of seven players—six court players and one goalie.

69. **Sports** The length of a regulation team handball court is twice its width.



a. Write a polynomial that represents the area of the court. $2x^2$

b. The width of a team handball court is 20 meters. Find the area of the court. 800 m²

Multiply.

70. $(1.5a^3)(4a^6)$

71. $(2x + 5)(x - 6)$

72. $(3g - 1)(g + 5)$

73. $(4x - 2y)(2x - 3y)$

74. $(x + 3)(x - 3)$

75. $(1.5x - 3)(4x + 2)$

76. $(x - 10)(x + 4)$

77. $x^2(x + 3)$

78. $(x + 1)(x^2 + 2x)$

79. $(x - 4)(2x^2 + x - 6)$

80. $(a + b)(a - b)^2$

81. $(2p - 3q)^3$

82. **Multi-Step** A rectangular swimming pool is 25 feet long and 10 feet wide. It is surrounded by a fence that is x feet from each side of the pool.

a. Draw a diagram of this situation.

b. Write expressions for the length and width of the fenced region. (*Hint:* How much longer is one side of the fenced region than the corresponding side of the pool?) $2x + 25$; $2x + 10$

c. Write an expression for the area of the fenced region. $4x^2 + 70x + 250$

83. **Write About It** Explain why the FOIL method can be used to multiply only two binomials at a time.

(cl), PHILIPPE DESMAZES/AFP/Getty Images; (tl), HMH

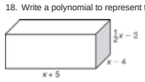
PRACTICE A

PRACTICE C

PRACTICE B

Multiply.

1. $(6m^4)(8m^2)$ — $48m^6$
2. $(5x^2)(4xy^2)$ — $20x^4y^2$
3. $(10s^5t)(7st^4)$ — $70s^6t^5$
4. $4(x^2 + 5x + 6)$ — $4x^2 + 20x + 24$
5. $2x(3x - 4)$ — $6x^2 - 8x$
6. $7xy(3x^2 + 4y + 2)$ — $21x^3y + 28xy^2 + 14xy$
7. $(x + 3)(x + 4)$ — $x^2 + 7x + 12$
8. $(x - 6)(x - 6)$ — $x^2 - 12x + 36$
9. $(x - 2)(x - 5)$ — $x^2 - 7x + 10$
10. $(2x + 5)(x + 6)$ — $2x^2 + 17x + 30$
11. $(m^2 + 3)(5m + n)$ — $5m^4 + m^3n + 15n + 3n$
12. $(a^2 + b^2)(a + b)$ — $a^3 + a^2b + ab^2 + b^3$
13. $(x + 4)(x^2 + 3x + 5)$ — $x^3 + 7x^2 + 17x + 20$
14. $(3m + 4)(m^2 - 3m + 5)$ — $3m^3 - 5m^2 + 3m + 20$
15. $(2x - 5)(4x^2 - 3x + 1)$ — $8x^3 - 26x^2 + 17x - 5$
16. **The length of a rectangle is 3 inches greater than the width.**
    a. Write a polynomial that represents the area of the rectangle. — $w^2 + 3w$
    b. Find the area of the rectangle when the width is 4 inches. — 28 in²
17. **The length of a rectangle is 8 centimeters less than 3 times the width.**
    a. Write a polynomial that represents the area of the rectangle. — $3w^2 - 8w$
    b. Find the area of the rectangle when the width is 10 centimeters. — 220 cm²
18. Write a polynomial to represent the volume of the rectangular prism.



$\frac{1}{2}x^3 - \frac{5}{2}x^2 - 13x + 60$

READING STRATEGIES

There are several methods that can be used to multiply polynomials, depending on the number of terms. There is one procedure that can always be used, no matter how many terms there are. It is shown in the example below.

Multiply $(5x - 4)(3x^2 + x - 8)$.

$(5x - 4)(3x^2 + x - 8)$
$5x(3x^2 + x - 8) - 4(3x^2 + x - 8)$ — 1 Use the Distributive Property.
$15x^3 + 5x^2 - 40x - 12x^2 - 4x + 32$
$15x^3 + 5x^2 - 40x - 12x^2 - 4x + 32$ — 2 Collect like terms.
$15x^3 + (5x^2 - 12x^2) + (-40x - 4x) + 32$
$15x^3 + (5x^2 - 12x^2) + (-40x - 4x) + 32$ — 3 Simplify by combining like terms.
$15x^3 - 7x^2 - 44x + 32$

Use the procedure shown above to answer each of the following.

1. Multiplication was used six times in step 1. How many times would it be used if two binomials were being multiplied? — 4
2. In step 2, how do you know that $5x^2$ and $-12x^2$ are like terms? — They have the same exponent on the same variable.
3. In step 3, how do you know the expression is completely simplified? — There are no like terms.

Multiply the polynomials.

4. $-3x^2(2x^2 - 4x + 1)$ — $-6x^5 + 12x^4 - 3x^3$
5. $(2x + 5)(9x^2 + 6x)$ — $18x^3 + 57x^2 + 30x$
6. $(7x + 2)(x - 3)$ — $7x^2 - 19x - 6$
7. $(2x^2 + 6x + 8)(x^2 - 5x + 1)$ — $2x^5 - 10x^4 + 8x^3 - 22x^2 - 34x + 8$

RETEACH

To multiply monomials, multiply the constants, then multiply variables with the same base.

Multiply $(3a^2b)(4ab^3)$.

$(3a^2b)(4ab^3)$
$(3 \cdot 4)(a^2 \cdot a)(b \cdot b^3)$ — *Rearrange so that the constants and the variables with the same bases are together.*
$12a^3b^4$ — *Multiply.*

To multiply a polynomial by a monomial, distribute the monomial to each term in the polynomial.

Multiply $2x(x^2 + 3x + 7)$.

$2x(x^2 + 3x + 7)$
$(2x)x^2 + (2x)3x + (2x)7$ — *Distribute.*
$2x^3 + 6x^2 + 14x$ — *Multiply.*

Multiply.

1. $(-5x^2y^2)(2xy)$ — $-10x^3y^4$
2. $(2xyz)(-4x^2yz)$ — $-8x^3y^2z^2$
3. $(3x)(x^2y^2)$ — $3x^3y^2$

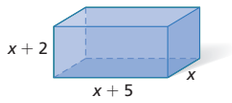
Fill in the blanks below. Then finish multiplying.

4. $4(x - 5)$ — $(\boxed{4})x - (\boxed{4})5$ — $4x - 20$
5. $3x(x + 8)$ — $(\boxed{3x})x + (\boxed{3x})8$ — $3x^2 + 24x$
6. $2x(x^2 - 6x + 3)$ — $(\boxed{2x})x^2 - (\boxed{2x})6x + (\boxed{2x})3$ — $2x^3 - 12x^2 + 6x$

Multiply.

7. $5(x + 9)$ — $5x + 45$
8. $-4x(x^2 + 8)$ — $-4x^3 - 32x$
9. $3x^2(2x^2 + 5x + 4)$ — $6x^4 + 15x^3 + 12x^2$
10. $-3(5 - x^2 + 2)$ — $3x^2 - 21$
11. $(5a^2b)(2ab)$ — $10a^4b^2$
12. $5y(-y^2 + 7y - 2)$ — $-5y^3 + 35y^2 - 10y$

84. Geometry Write a polynomial that represents the volume of the rectangular prism. $x^3 + 7x^2 + 10x$



85. Critical Thinking Is there any value for x that would make the statement $(x+3)^3 = x^3 + 3^3$ true? Give an example to justify your answer. Yes; $x = 0$

86. Estimation The length of a rectangle is 1 foot more than its width. Write a polynomial that represents the area of the rectangle. Estimate the width of the rectangle if its area is 25 square feet. $x^2 + x$; 4.5 ft

TEST PREP

87. Which of the following products is equal to $a^2 - 5a - 6$?

Ⓐ $(a-1)(a-5)$ Ⓑ $(a-2)(a-3)$ Ⓒ $(a+1)(a-6)$ Ⓓ $(a+2)(a-3)$

88. Which of the following is equal to $2a(a^2 - 1)$?

Ⓕ $2a^2 - 2a$ Ⓖ $2a^3 - 1$ Ⓗ $2a^3 - 2a$ Ⓙ $2a^2 - 1$

89. What is the degree of the product of $3x^3y^2z$ and x^2yz?

Ⓐ 5 Ⓑ 6 Ⓒ 7 Ⓓ 10

CHALLENGE AND EXTEND

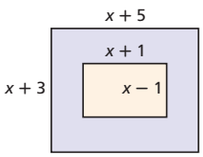
Simplify.

90. $6x^2 - 2(3x^2 - 2x + 4)$ $4x - 8$

91. $x^2 - 2x(x+3)$ $-x^2 - 6x$

92. $x(4x-2) + 3x(x+1)$ $7x^2 + x$

93. The diagram shows a sandbox and the frame that surrounds it.



a. Write a polynomial that represents the area of the sandbox. $x^2 - 1$

b. Write a polynomial that represents the area of the frame that surrounds the sandbox. $8x + 16$

94. Geometry The side length of a square is $(8 + 2x)$ units. The area of this square is the same as the perimeter of another square with a side length of $(x^2 + 48)$ units. Find the value of x. $x = 4$

95. Write a polynomial that represents the product of three consecutive integers. Let x represent the first integer. $x^3 + 3x^2 + 2x$

96. Find m and n so that $x^m(x^n + x^{n-2}) = x^5 + x^3$. Possible answer: $m = 2$; $n = 3$

97. Find a so that $2x^a(5x^{2a-3} + 2x^{2a+2}) = 10x^3 + 4x^8$ $a = 2$

PROBLEM SOLVING

Write the correct answer.

1. A bedroom has a length of $x + 3$ feet and a width of $x - 1$ feet. Write a polynomial to express the area of the bedroom. Then calculate the area if $x = 10$.
   $x^2 + 2x - 3$
   117 square feet

2. The length of a classroom is 4 feet longer than its width. Write a polynomial to express the area of the classroom. Then calculate the area if the width is 22 feet.
   $w^2 + 4w$
   572 square feet

3. Nicholas is determining if he can afford to buy a car. He multiplies the number of months m by $i + p + 30f$ where i represents the monthly cost of insurance, p represents the monthly car payment, and f represents the number of times he fills the gas tank each month. Write the polynomial that Nicholas can use to determine how much it will cost him to own a car both for one month and for one year.
   $i + p + 30f$; $12i + 12p + 360f$

4. A seat cushion is shaped like a trapezoid. The shorter base of the cushion is 3 inches greater than the height. The longer base is 2 inches shorter than twice the height. Write the polynomial that can be used to find the area of the cushion. (The area of a trapezoid is represented by $\frac{1}{2}h(b_1 + b_2)$.)
   $\frac{3}{2}h^2 + \frac{1}{2}h$

The volume of a pyramid can be found by using $\frac{1}{3}Bh$ where B is the area of the base and h is the height of the pyramid. The Great Pyramid of Giza has a square base, and each side is about 300 feet longer than the height of the pyramid. Select the best answer.

5. Which polynomial represents the approximate area of the base of the Great Pyramid?
   A $h + 90{,}000$
   B $2h + 90{,}000$
   C $h^2 + 600h + 90{,}000$
   D $2h^2 + 600h + 90{,}000$

6. Which polynomial represents the approximate volume of the Great Pyramid?
   F $\frac{1}{3}h^3 + 200h^2 + 30{,}000h$
   G $\frac{1}{3}h^2 + 200h + 30{,}000$
   H $h^3 + 600h^2 + 90{,}000h$
   J $3h^3 + 600h^2 + 90{,}000h$

7. The original height of the Great Pyramid was 485 feet. Due to erosion, it is now about 450 feet. Find the approximate volume of the Great Pyramid today.
   A 562,500 ft³
   B 616,225 ft³
   C 84,375,000 ft³
   D 99,623,042 ft³

CHALLENGE

Determine the missing binomial. Choose from the table below. Each binomial is used once.

$x + 4$	$x + 3$	$x - 3$
$x + 1$	$x + 6$	$x - 4$
$x + 1$	$x - 2$	$x - 6$

1. $(x+4)(x+2) = x^2 + 2x + 4x + 8 = x^2 + 6x + 8$
2. $(x+1)(x+5) = x^2 + 5x + x + 5 = x^2 + 6x + 5$
3. $(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$
4. $(x+3)(x-5) = x^2 - 2x - 15$
5. $(x+6)(x+1) = x^2 + 7x + 6$
6. $(x-2)(x+4) = x^2 + 2x - 8$
7. $(x-3)(x+6) = x^2 + 3x - 18$
8. $(x-4)(x+2) = x^2 - 2x - 8$
9. $(x-6)(x-5) = x^2 - 11x + 30$

The binomials missing from the following equations are not all listed in the table. Determine the missing binomials.

10. $(x+2)(x+7) = x^2 + 9x + 14$
11. $(x+3)(x+4) = x^2 + 7x + 12$
12. $(x-6)(x-2) = x^2 - 8x + 12$
13. $(x-4)(x-10) = x^2 - 14x + 40$
14. $(x+6)(x+3) = x^2 + 9x + 18$
15. $(x+1)(x-4) = x^2 - 3x - 4$
16. $(x-2)(x-2) = x^2 - 4x + 4$
17. $(3x-1)(x+2) = 3x^2 + 5x - 2$
18. $(x-7)(2x+3) = 2x^2 - 11x - 21$
19. $(2x-3)(3x+4) = 6x^2 - x - 12$

TEST PREP DOCTOR Because the last term of the polynomial in **Exercise 87** is negative, the signs of the second terms in the binomials must be different. Eliminate choices **A** and **B.**

Multiplying the first terms in **Exercise 88** gives $2a^3$, so choices **F** and **J** cannot be correct. Students who chose **G** probably did not distribute fully.

Journal

Have students explain how to multiply $(2x + 3)$ by $(x + 4)$ using the method of their choice.

ALTERNATIVE ASSESSMENT

Have students count off by fours. Have the 1's write a monomial, the 2's write a binomial, the 3's write a trinomial, and the 4's write a polynomial of degree four. Rearrange the class into new groups of four, so that each new group has a monomial, a binomial, a trinomial, and a 4th-degree polynomial. (Make some groups of five students if needed.) Have students multiply pairs of their polynomials together. Each group must turn in as many pairs as possible, with work shown for each answer.

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6-5 Lesson Quiz

Multiply.

1. $(6s^2t^2)(3st)$ $18s^3t^3$
2. $4xy^2(x + y)$ $4x^2y^2 + 4xy^3$
3. $(x + 2)(x - 8)$ $x^2 - 6x - 16$
4. $(2x - 7)(x^2 + 3x - 4)$ $2x^3 - x^2 - 29x + 28$
5. $6mn(m^2 + 10mn - 2)$ $6m^3n + 60m^2n^2 - 12mn$
6. $(2x - 5y)(3x + y)$ $6x^2 - 13xy - 5y^2$
7. A triangle has a base that is 4 cm longer than its height.
   a. Write a polynomial that represents the area of the triangle. $\frac{1}{2}h^2 + 2h$
   b. Find the area when the height is 8 cm. 48 cm²

Also available online

Organizer

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Apply polynomial operations to finding areas of geometric figures.

Online Edition

Countdown

Teach

Remember

Students review and apply volume and surface area formulas for geometric figures.

Close

Assess

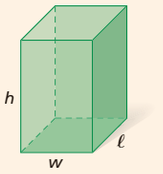
The volume of any prism equals the area of the base times the height. Have students explain how to find the volume of the triangular prism in **Problem 6.** Find the area of the base. It is $2y^2 + y$. Multiply this by the height, which is $3y$. The volume is $6y^3 + 3y^2$.

Learn It Online
State Resources Online

Volume and Surface Area

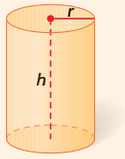
The volume V of a three-dimensional figure is the amount of space it occupies. The surface area S is the total area of the two-dimensional surfaces that make up the figure.

Rectangular Prism



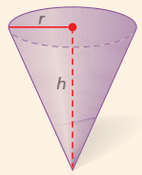
$V = \ell wh$
$S = 2(\ell w + \ell h + wh)$

Cylinder



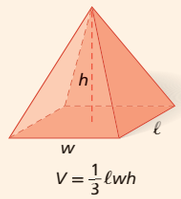
$V = \pi r^2 h$
$S = 2\pi r^2 + 2\pi rh$

Cone



$V = \frac{1}{3}\pi r^2 h$

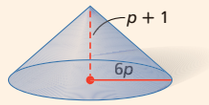
Pyramid



$V = \frac{1}{3}\ell wh$

Example

Write and simplify a polynomial expression for the volume of the cone. Leave the symbol π in your answer.



$V = \frac{1}{3}\pi r^2 h$ *Choose the correct formula.*

$= \frac{1}{3}\pi(6p)^2(p+1)$ *Substitute 6p for r and p + 1 for h.*

$= \frac{1}{3}\pi(36p^2)(p+1)$ *Use the Power of a Product Property.*

$= \frac{1}{3}(36)\pi[p^2(p+1)]$ *Use the Associative and Commutative Properties of Multiplication.*

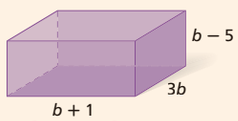
$= 12\pi p^2(p+1)$ *Distribute $12\pi p^2$.*

$= 12\pi p^3 + 12\pi p^2$

Try This

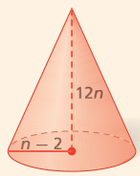
Write and simplify a polynomial expression for the volume of each figure.

1.



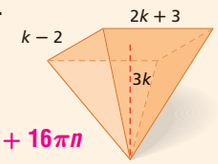
$3b^3 - 12b^2 - 15b$

2.



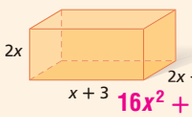
$4\pi n^3 - 16\pi n^2 + 16\pi n$

3. $2k^3 - k^2 - 6k$



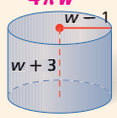
Write and simplify a polynomial expression for the surface area of each figure.

4.

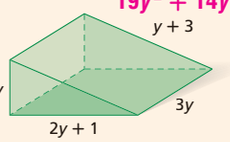


$16x^2 + 30x + 6$

5. $4\pi w^2 - 4\pi$



6. $19y^2 + 14y$



EXTENSION

Closure

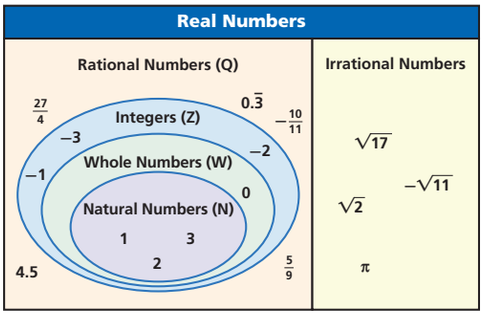
CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *Also* **CC.9-12.N.RN.3**

Objective
Identify sets and the operations under which they are closed.

Vocabulary
set
element
subset
closure

A **set** is a collection of objects. Each object in a set is called an **element** of the set. A set may have no elements, a finite number of elements, or an infinite number of elements. For example, $N = \{1, 2, 3, \ldots\}$ describes the set of natural numbers.

A **subset** is a set contained entirely within another set. For example, $A = \{2, 6, 11, 50\}$ is a subset of set N above. Also, N is a subset of the set of real numbers. The diagram below shows other subsets of the real numbers.



A set of numbers is *closed*, or has **closure**, under a given operation if the result of the operation on any two numbers in the set is also in the set. For example, the set of even numbers is closed under addition, since the sum of two even numbers is also an even number.

Closure Properties of the Real Numbers

WORDS	NUMBERS	ALGEBRA
The real numbers are closed under addition, subtraction, and multiplication.	$6.1 + \sqrt{2}$, $6.1 - \sqrt{2}$, and $6.1 \times \sqrt{2}$ are all real numbers.	For real numbers a and b, $a + b$, $a - b$, and ab are real numbers.

EXAMPLE 1 **Determining Closure of Sets of Numbers**

A **Is the set $\{-1, 0, 1\}$ closed under multiplication?**

Multiply each pair of elements in the set. Check whether each product is in the set.

$-1 \times (-1) = 1$ ✓ $\quad -1 \times 1 = -1$ ✓ $\quad 0 \times 1 = 0$ ✓
$-1 \times 0 = 0$ ✓ $\quad 0 \times 0 = 0$ ✓ $\quad 1 \times 1 = 1$ ✓

The set $\{-1, 0, 1\}$ is closed under multiplication.

B **Show that the set of irrational numbers is not closed under multiplication.**

Find two irrational numbers whose product is not an irrational number.

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$ *2 is not irrational.*

The set of irrational numbers is not closed under multiplication.

Helpful Hint

Remember the Commutative Property of Multiplication.

If $1 \times (-1) = -1$, then $-1 \times 1 = -1$. Only one instance needs to be tested.

CHAPTER 6 **Organizer**

Pacing: Traditional 2 days
Block 1 day

Objective: Identify sets and the operations under which they are closed.

Online Edition

Using the Extension

In this chapter, students learned about polynomials and exponents and performing operations with polynomials. After learning how to multiply polynomials, students can use this extension to learn about closure. The extension will help students understand sets of numbers and determine whether or not a set of numbers or a set of polynomials is closed under a given operation.

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
CC.9-12.N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Learn It Online
State Resources Online

1 Introduce

Motivate

Ask students to determine whether the first set of numbers is included in the second set of numbers and explain.

1. integers, whole numbers no; neg. integers are not whole numbers.
2. natural numbers, whole numbers yes; whole numbers include all natural numbers and 0.
3. fractions, real numbers yes; all fractions are real numbers.

2 Teach

Guided Instruction

When working with closure, students need to understand sets and subsets. Use the Venn diagram of real numbers to show students how one group of numbers can be contained within a second group of numbers. Then explain how an operation performed on two numbers in a set may result in a number that is part of the original set. If this is the case, then the set is closed under that operation.

COMMON ERROR ALERT

Students may forget to verify an operation using a single element of a set twice. For example, the set {0, 3} would appear to be closed if only 0 + 3 is checked. However, 3 + 3 = 6, so the set is not closed under addition.

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Additional Examples

Example 1

A. Is the set {−2, 0, 2} closed under multiplication? no; $-2 \cdot 2 = -4$, which is not part of the original set.

B. Show that the set of irrational numbers is not closed under division. Possible answer: $\frac{\sqrt{2}}{\sqrt{2}} = 1$, which is not an irrational number.

Example 2

A. Is the set {0, 1, x, $x + 1$} closed under addition? Explain. no; $x + x + 1 = 2x + 1$, which is not a member of the set.

B. Show that the set of polynomials is closed under multiplication. Since each product of two polynomials will result in a polynomial, the set is closed.

Also available online

Questioning Strategies

EXAMPLE 1

- How can you determine if a subset of the real numbers is closed under a given operation?

EXAMPLE 2

- How can you show that the set of polynomials is closed under addition?

1. Possible answer: 4 − 5 = −1, which is not a whole number.

CHECK IT OUT! **1.** Show that the set of whole numbers is not closed under subtraction.

The set of polynomials also has closure under certain operations. The closure properties of polynomials are similar to the closure properties of real numbers.

Closure Properties of Polynomials

WORDS	EXAMPLES	ALGEBRA
The set of all polynomials is closed under addition, subtraction, and multiplication.	$x^2 + (x + 1)$, $x^2 - (x + 1)$ and $x^2(x + 1)$ are all polynomials.	For polynomials p and q, $p + q$, $p - q$, and pq are all polynomials.

EXAMPLE 2 **Determining Closure of Sets of Polynomials**

A **Is the set {−x, 0, x} closed under addition?**

Add each pair of elements in the set. Check whether each sum is in the set.

$-x + 0 = -x$ ✓ $\quad -x + (-x) = -2x$ ✗ $\quad 0 + 0 = 0$ ✓

$-x + x = 0$ ✓ $\quad 0 + x = x$ ✓ $\quad x + x = 2x$ ✗

The set {−x, 0, x} is not closed under addition.

B **Show that the set of polynomials is not closed under division.**

Find two polynomials, a and b, such that their quotient, $\frac{a}{b}$, is not a polynomial. Try $a = x + 1$ and $b = 0$.

$\frac{x+1}{0}$ = undefined *The result is not a polynomial.*

The set of polynomials is not closed under division.

CHECK IT OUT! **2.** Is the set {x, $x + 1$, $x^2 - 1$} closed under division? no

EXTENSION Exercises

16. The rational numbers are closed under +, −, and ×. The irrational numbers are not closed under any of the 4 ops.

Determine whether the following sets are closed under addition, subtraction, multiplication, and division.

1. {−1, 0, 1} ×
2. {0, 8} none
3. {x^2, 1} none
4. {0, x} none
5. {$-x^3$, 1, x^3} none
6. {−x, 1, $x + 1$} none
7. {−1, 1} ×, ÷
8. {−1, 0, x} none
9. {−1, $x + 3$, 1} none
10. The set of whole numbers +, ×
11. The set of natural numbers +, ×
12. The set of integers +, −, ×
13. Polynomials without a constant term +, −, ×
14. The set of rational numbers +, −, ×
15. The set of real numbers +, −, ×
16. **Write About It** Compare closure properties under the four operations for the set of rational numbers and the set of irrational numbers.

3 Close

Summarize

A set is closed under an operation if the result of the operation on any two numbers in the set is an element of the original set. Remind students that the set of real numbers is closed under addition, subtraction, and multiplication, but not all subsets of the real numbers are closed under these same operations.

COMMON CORE

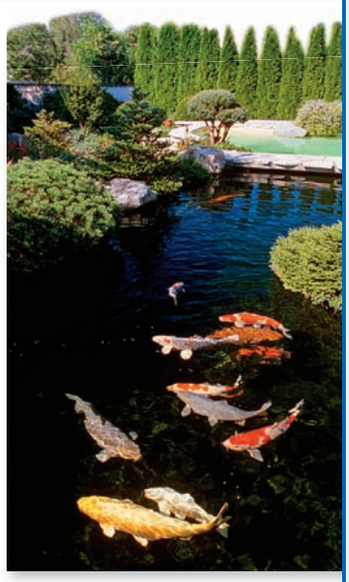
6-6 Special Products of Binomials

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

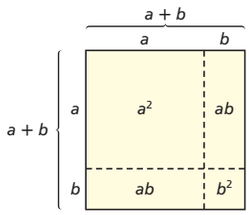
Objective
Find special products of binomials.

Vocabulary
perfect-square trinomial
difference of two squares

Why learn this?
You can use special products to find areas, such as the area of a deck around a pond. (See Example 4.)



Imagine a square with sides of length $(a + b)$:



The area of this square is $(a + b)(a + b)$, or $(a + b)^2$. The area of this square can also be found by adding the areas of the smaller squares and rectangles inside. The sum of the areas inside is $a^2 + ab + ab + b^2$.

This means that $(a + b)^2 = a^2 + 2ab + b^2$.

You can use the FOIL method to verify this:

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

F L I O

A trinomial of the form $a^2 + 2ab + b^2$ is called a *perfect-square trinomial*. A **perfect-square trinomial** is a trinomial that is the result of squaring a binomial.

EXAMPLE 1 Finding Products in the Form $(a + b)^2$

Multiply.

A $(x + 4)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a + b)^2$.*

$(x + 4)^2 = x^2 + 2(x)(4) + 4^2$ — *Identify a and b: a = x and b = 4.*

$= x^2 + 8x + 16$ — *Simplify.*

B $(3x + 2y)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a + b)^2$.*

$(3x + 2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$ — *Identify a and b: a = 3x and b = 2y.*

$= 9x^2 + 12xy + 4y^2$ — *Simplify.*

Juniors Bildarchiv/Alamy

6-6 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Find special products of binomials.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. 4^2 16
2. 7^2 49
3. $(-2)^2$ 4
4. $(x)^2$ x^2
5. $-(5y^2)$ $-5y^2$
6. $(m^2)^2$ m^4
7. $2(6xy)$ $12xy$
8. $2(8x^2)$ $16x^2$

Also available online

Math Humor

Parent: What happened in math class today?

Student: When the teacher said to look for perfect squares, everyone looked at me.

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Special Products of Binomials

You can use the FOIL method to find the square of a binomial. For example, to find $(a + 1)^2$, first write the product as $(a + 1)(a + 1)$. Then use the FOIL method.

$$(a + 1)^2 = (a + 1)(a + 1)$$
$$= a^2 + a \cdot 1 + 1 \cdot a + 1^2$$
$$= a^2 + a + a + 1$$
$$= a^2 + 2a + 1$$

1. Use the FOIL method to complete the table.

Power	Expanded Form	Product
$(a + 1)^2$	$(a + 1)(a + 1)$	$a^2 + 2a + 1$
$(a + 2)^2$	$(a + 2)(a + 2)$	$a^2 + 4a + 4$
$(a + 3)^2$	$(a + 3)(a + 3)$	$a^2 + 6a + 9$
$(a + 4)^2$	$(a + 4)(a + 4)$	$a^2 + 8a + 16$

2. Look for a pattern in the right column of the table. Use the pattern to find $(a + 5)^2$ without using the FOIL method.
3. Find $(a + 9)^2$ without using the FOIL method.

THINK AND DISCUSS

4. **Describe** a general rule you can use to find $(a + b)^2$.
5. **Show** how you can apply your rule to find $(7 + x)^2$.

Motivate

Have students find the products of the following:

$(x + 3)(x + 3)$ $x^2 + 6x + 9$
$(x + 4)(x + 4)$ $x^2 + 8x + 16$
$(x + 5)(x + 5)$ $x^2 + 10x + 25$

Discuss with students any patterns they see. Lead them to recognize that the middle term of the trinomial is two times the product of the first and last terms of the binomial.

$(a + b)(a + b) = a^2 + 2ab + b^2$

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

Multiply.

A. $(x + 3)^2$ $x^2 + 6x + 9$

B. $(4s + 3t)^2$ $16s^2 + 24st + 9t^2$

C. $(5 + m^2)^2$ $25 + 10m^2 + m^4$

Example 2

Multiply.

A. $(x - 6)^2$ $x^2 - 12x + 36$

B. $(4m - 10)^2$ $16m^2 - 80m + 100$

C. $(2x - 5y)^2$ $4x^2 - 20xy + 25y^2$

D. $(7 - r^3)^2$ $49 - 14r^3 + r^6$

Also available online

Multiply.

C $(4 + s^2)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(4 + s^2)^2 = (4)^2 + 2(4)(s^2) + (s^2)^2$ *Identify a and b: a = 4 and b = s^2.*

$= 16 + 8s^2 + s^4$ *Simplify.*

D $(-m + 3)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(-m + 3)^2 = (-m)^2 + 2(-m)(3) + 3^2$ *Identify a and b: a = −m and b = 3.*

$= m^2 - 6m + 9$ *Simplify.*

CHECK IT OUT! **Multiply.**

1a. $(x + 6)^2$ $x^2 + 12x + 36$

1b. $(5a + b)^2$ $25a^2 + 10ab + b^2$

1c. $(1 + c^3)^2$ $1 + 2c^3 + c^6$

You can use the FOIL method to find products in the form $(a - b)^2$:

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2$$

(F, L, I, O)

$$= a^2 - 2ab + b^2$$

A trinomial of the form $a^2 - 2ab + b^2$ is also a perfect-square trinomial because it is the result of squaring the binomial $(a - b)$.

EXAMPLE 2 Finding Products in the Form $(a - b)^2$

Multiply.

A $(x - 5)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(x - 5)^2 = x^2 - 2(x)(5) + 5^2$ *Identify a and b: a = x and b = 5.*

$= x^2 - 10x + 25$ *Simplify.*

B $(6a - 1)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(6a - 1)^2 = (6a)^2 - 2(6a)(1) + (1)^2$ *Identify a and b: a = 6a and b = 1.*

$= 36a^2 - 12a + 1$ *Simplify.*

C $(4c - 3d)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(4c - 3d)^2 = (4c)^2 - 2(4c)(3d) + (3d)^2$ *Identify a and b: a = 4c and b = 3d.*

$= 16c^2 - 24cd + 9d^2$ *Simplify.*

D $(3 - x^2)^2$

$(a - b)^2 = (a)^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(3 - x^2)^2 = (3)^2 - 2(3)(x^2) + (x^2)^2$ *Identify a and b: a = 3 and b = x^2.*

$= 9 - 6x^2 + x^4$ *Simplify.*

2a. $x^2 - 14x + 49$
2b. $9b^2 - 12bc + 4c^2$
2c. $a^4 - 8a^2 + 16$

CHECK IT OUT! **Multiply.**

2a. $(x - 7)^2$ **2b.** $(3b - 2c)^2$ **2c.** $(a^2 - 4)^2$

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you find the middle term?
- In which position could a perfect-square trinomial have a term with an odd exponent?

EXAMPLE 2

- How are these examples different from those in **Example 1**? What causes the differences? How are they the same?
- Why is the last term of any perfect-square trinomial always positive?

2 Teach

Guided Instruction

Review how to multiply any two binomials before starting this section. Show the FOIL method for $(a + b)^2$ and $(a - b)^2$ next to each other, so students can better visualize the similarities and differences. Tell students that the binomials in this lesson could be multiplied using any of the methods they already know, but that certain types of binomials can be multiplied more quickly knowing these rules.

Reaching All Learners

Through Auditory Cues

Have students learn the "verbal rules" for the special products.

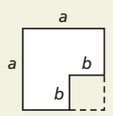
$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

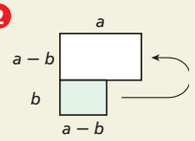
- first term squared
- plus (or minus) two times the product of both terms
- plus last term squared

Have students create a similar verbal rule for $(a + b)(a - b) = a^2 - b^2$.

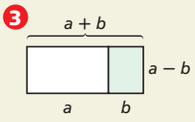
You can use an area model to see that $(a + b)(a - b) = a^2 - b^2$.



Begin with a square with area a^2. Remove a square with area b^2. The area of the new figure is $a^2 - b^2$.



Then remove the smaller rectangle on the bottom. Turn it and slide it up next to the top rectangle.



The new arrangement is a rectangle with length $a + b$ and width $a - b$. Its area is $(a + b)(a - b)$.

So $(a + b)(a - b) = a^2 - b^2$. A binomial of the form $a^2 - b^2$ is called a **difference of two squares**.

EXAMPLE 3 Finding Products in the Form $(a + b)(a - b)$

Multiply.

A $(x + 6)(x - 6)$

$(a + b)(a - b) = a^2 - b^2$ — *Use the rule for $(a + b)(a - b)$.*

$(x + 6)(x - 6) = x^2 - 6^2$ — *Identify a and b: a = x and b = 6.*

$= x^2 - 36$ — *Simplify.*

B $(x^2 + 2y)(x^2 - 2y)$

$(a + b)(a - b) = a^2 - b^2$ — *Use the rule for $(a + b)(a - b)$.*

$(x^2 + 2y)(x^2 - 2y) = (x^2)^2 - (2y)^2$ — *Identify a and b: $a = x^2$ and $b = 2y$.*

$= x^4 - 4y^2$ — *Simplify.*

C $(7 + n)(7 - n)$

$(a + b)(a - b) = a^2 - b^2$ — *Use the rule for $(a + b)(a - b)$.*

$(7 + n)(7 - n) = 7^2 - n^2$ — *Identify a and b: a = 7 and b = n.*

$= 49 - n^2$ — *Simplify.*

CHECK IT OUT! **Multiply.**

3a. $(x + 8)(x - 8)$ $x^2 - 64$

3b. $(3 + 2y^2)(3 - 2y^2)$ $9 - 4y^4$

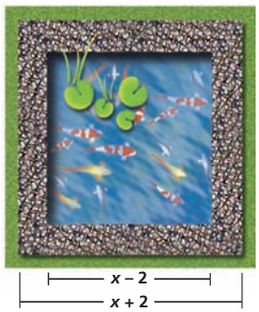
3c. $(9 + r)(9 - r)$ $81 - r^2$

EXAMPLE 4 *Problem-Solving Application*

A square koi pond is surrounded by a gravel path. Write an expression that represents the area of the path.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.



1 Understand the Problem

The **answer** will be an expression that represents the area of the path.

List the important information:

- The pond is a square with a side length of $x - 2$.
- The path has a side length of $x + 2$.

Sam Dudgeon/HMH

Teaching Tip **Math Background** The binomials $(a + b)$ and $(a - b)$ are called *conjugates*. Conjugates are used in many situations because of the special property that their product is always a difference of squares.

COMMON ERROR ALERT

When finding the product of $(a - b)^2$, some students might believe that the middle term and the last term will both be negative numbers. Remind them that the last term comes from a number being squared, and thus will never be a negative.

Power Presentations with PowerPoint®

Additional Examples

Example 3

Multiply.

A. $(x + 4)(x - 4)$ $x^2 - 16$

B. $(p^2 + 8q)(p^2 - 8q)$ $p^4 - 64q^2$

C. $(10 + b)(10 - b)$ $100 - b^2$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- Why is there no middle term in the product $(a + b)(a - b)$?
- Why does the product $(a + b)(a - b)$ always have a minus sign between the terms?

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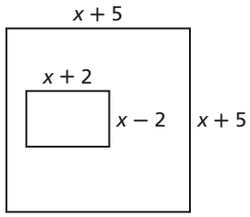
Additional Examples

Example 4

Write a polynomial that represents the area of the yard around the pool shown below. $10x + 29$



Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- What are you finding when you are multiplying the binomials?
- What area are you asked to find?
- What operation would you need to perform to get this result?

2 Make a Plan

The area of the pond is $(x - 2)^2$. The total area of the path plus the pond is $(x + 2)^2$. You can subtract the area of the pond from the total area to find the area of the path.

3 Solve

Step 1 Find the total area.

$(x + 2)^2 = x^2 + 2(x)(2) + 2^2$ *Use the rule for $(a + b)^2$: $a = x$ and $b = 2$.*

$= x^2 + 4x + 4$

Step 2 Find the area of the pond.

$(x - 2)^2 = x^2 - 2(x)(2) + 2^2$ *Use the rule for $(a - b)^2$: $a = x$ and $b = 2$.*

$= x^2 - 4x + 4$

Step 3 Find the area of the path.

area of path = total area − area of pond

$a = x^2 + 4x + 4 - (x^2 - 4x + 4)$

$= x^2 + 4x + 4 - x^2 + 4x - 4$ *Identify like terms.*

$= (x^2 - x^2) + (4x + 4x) + (4 - 4)$ *Group like terms together.*

$= 8x$

The area of the path is $8x$. *Combine like terms.*

4 Look Back

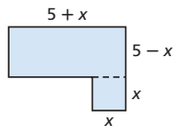
Suppose that $x = 10$. Then one side of the path is 12, and the total area is 12^2, or 144. Also, if $x = 10$, one side of the pond is 8, and the area of the pond is 8^2, or 64. This means the area of the path is $144 - 64 = 80$.

According to the solution above, the area of the path is $8x$. If $x = 10$, then $8x = 8(10) = 80$. ✓

Remember!

To subtract a polynomial, add the opposite of *each* term.

CHECK IT OUT! **4.** Write an expression that represents the area of the swimming pool at right. **25**



Know it! Note

Special Products of Binomials

Perfect-Square Trinomials

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Difference of Two Squares

$(a + b)(a - b) = a^2 - b^2$

3 Close

Summarize

Have students state whether each product is a perfect-square trinomial or a difference of two squares, and then find the product.

$(2x + 3y)^2$ perfect-square trinomial; $4x^2 + 12xy + 9y^2$

$(5m + 7)(5m - 7)$ difference of two squares; $25m^2 - 49$

$(4s - t)^2$ perfect-square trinomial; $16s^2 - 8st + t^2$

Tell students that recognizing special products will greatly help them with factoring.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Use the FOIL method to verify that $(a + b)(a - b) = a^2 - b^2$.
2. When a binomial is squared, the middle term of the resulting trinomial is twice the _____?_____ of the first and last terms.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Complete the special product rules and give an example of each.

Special Products of Binomials		
Perfect-Square Trinomials		Difference of Two Squares
$(a + b)^2 = ?$	$(a - b)^2 = ?$	$(a + b)(a - b) = ?$

6-6 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** In your own words, describe a *perfect-square trinomial*.
Possible answer: a trinomial that is the result of squaring a binomial

Multiply.

SEE EXAMPLE 1

2. $(x + 7)^2$
3. $(2 + x)^2$
4. $(x + 1)^2$
5. $(2x + 6)^2$
6. $(5x + 9)^2$
7. $(2a + 7b)^2$

SEE EXAMPLE 2

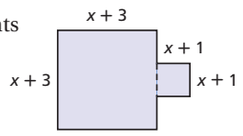
8. $(x - 6)^2$
9. $(x - 2)^2$
10. $(2x - 1)^2$
11. $(8 - x)^2$
12. $(6p - q)^2$
13. $(7a - 2b)^2$

SEE EXAMPLE 3

14. $(x + 5)(x - 5)$
15. $(x + 6)(x - 6)$
16. $(5x + 1)(5x - 1)$
17. $(2x^2 + 3)(2x^2 - 3)$
18. $(9 - x^3)(9 + x^3)$
19. $(2x - 5y)(2x + 5y)$

SEE EXAMPLE 4

20. **Geometry** Write a polynomial that represents the area of the figure. $2x^2 + 8x + 10$



PRACTICE AND PROBLEM SOLVING

Multiply.

21. $(x + 3)^2$
22. $(4 + z)^2$
23. $(x^2 + y^2)^2$
24. $(p + 2q^3)^2$
25. $(2 + 3x)^2$
26. $(r^2 + 5t)^2$
27. $(s^2 - 7)^2$
28. $(2c - d^3)^2$
29. $(a - 8)^2$
30. $(5 - w)^2$
31. $(3x - 4)^2$
32. $(1 - x^2)^2$
33. $(a - 10)(a + 10)$
34. $(y + 4)(y - 4)$
35. $(7x + 3)(7x - 3)$
36. $(x^2 - 2)(x^2 + 2)$
37. $(5a^2 + 9)(5a^2 - 9)$
38. $(x^3 + y^2)(x^3 - y^2)$

Independent Practice

For Exercises	See Example
21–26	1
27–32	2
33–38	3
39	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Answers

2. $x^2 + 14x + 49$
3. $4 + 4x + x^2$
4. $x^2 + 2x + 1$
5. $4x^2 + 24x + 36$
6. $25x^2 + 90x + 81$
7. $4a^2 + 28ab + 49b^2$
8. $x^2 - 12x + 36$
9. $x^2 - 4x + 4$
10. $4x^2 - 4x + 1$
11. $64 - 16x + x^2$
12. $36p^2 - 12pq + q^2$
13. $49a^2 - 28ab + 4b^2$
14. $x^2 - 25$
15. $x^2 - 36$
16. $25x^2 - 1$
17. $4x^4 - 9$
18. $81 - x^6$
19. $4x^2 - 25y^2$
21. $x^2 + 6x + 9$
22. $16 + 8z + z^2$
23. $x^4 + 2x^2y^2 + y^4$
24. $p^2 + 4pq^3 + 4q^6$
25. $4 + 12x + 9x^2$
26. $r^4 + 10r^2t + 25t^2$
27. $s^4 - 14s^2 + 49$
28. $4c^2 - 4cd^3 + d^6$
29. $a^2 - 16a + 64$
30. $25 - 10w + w^2$
31. $9x^2 - 24x + 16$
32. $1 - 2x^2 + x^4$
33. $a^2 - 100$
34. $y^2 - 16$
35. $49x^2 - 9$
36. $x^4 - 4$
37. $25a^4 - 81$
38. $x^6 - y^4$

Answers to *Think and Discuss*

1. $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$
2. product
3. See Additional Answers.

6-6 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 21–32, 53–57
Average 21–32, 53–57
Advanced 21–32, 41, 44–48 even, 54, 56

If you finished Examples **1–4**
Basic 21–63, 64, 67–70
Average 21–40, 42–52 even, 53–73
Advanced 21–52, 58–62, 64–74

Homework Quick Check
Quickly check key concepts.
Exercises: 22, 28, 36, 39, 42

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 39–40, 61, 64

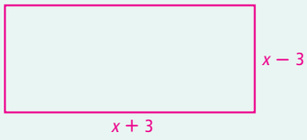
Construct viable arguments and critique the reasoning of others. Exercises 62–63, 65–66

Look for and make use of structure. Exercises 53–60, 74

Learn It Online
State Resources Online

Answers

64a.



39. $\pi x^2 + 8\pi x + 16\pi$

40a. $x > 2$; values less than or equal to 2 cause the width of the rectangle to be zero or neg., which does not make sense.

39. Entertainment Write a polynomial that represents the area of the circular puzzle. Remember that the formula for area of a circle is $A = \pi r^2$, where r is the radius of the circle. Leave the symbol π in your answer.

40. Multi-Step A square has sides that are $(x - 1)$ units long and a rectangle has a length of x units and a width of $(x - 2)$ units.

a. What are the possible values of x? Explain.

b. Which has the greater area, the square or the rectangle? square

c. What is the difference in the areas? 1 sq. unit

Multiply.

41. $(x + y)^2$ $x^2 + 2xy + y^2$

42. $(x - y)^2$ $x^2 - 2xy + y^2$

43. $(x^2 + 4)(x^2 - 4)$ $x^4 - 16$

44. $(x^2 + 4)^2$ $x^4 + 8x^2 + 16$

45. $(x^2 - 4)^2$ $x^4 - 8x^2 + 16$

46. $(1 - x)^2$ $1 - 2x + x^2$

49. $x^6 - 2a^3x^3 + a^6$

47. $(1 + x)^2$ $1 + 2x + x^2$

48. $(1 - x)(1 + x)$ $1 - x^2$

49. $(x^3 - a^3)(x^3 - a^3)$

50. $(5 + n)(5 + n)$ $25 + 10n + n^2$

51. $(6a - 5b)(6a + 5b)$ $36a^2 - 25b^2$

52. $(r - 4t^4)(r - 4t^4)$ $r^2 - 8rt^4 + 16t^8$

Copy and complete the tables to verify the special products of binomials.

	a	b	$(a - b)^2$	$a^2 - 2ab + b^2$
	1	4	$(1 - 4)^2 = 9$	$1^2 - 2(1)(4) + 4^2 = 9$
53.	2	4	4	4
54.	3	2	1	1

	a	b	$(a + b)^2$	$a^2 + 2ab + b^2$
55.	1	4	25	25
56.	2	5	49	49
57.	3	0	9	9

	a	b	$(a + b)(a - b)$	$a^2 - b^2$
58.	1	4	−15	−15
59.	2	3	−5	−5
60.	3	2	5	5

LINK Math History

Beginning about 3000 B.C.E., the Babylonians lived in what is now Iraq and Turkey. Around 575 B.C.E., they built the Ishtar Gate to serve as one of eight main entrances into the city of Babylon. The image above is a relief sculpture from a restoration of the Ishtar Gate.

61. Math History The Babylonians used tables of squares and the formula $ab = \frac{(a + b)^2 - (a - b)^2}{4}$ to multiply two numbers. Use this formula to find the product $35 \cdot 24$. 840

62. Critical Thinking Find a value of c that makes $16x^2 - 24x + c$ a perfect-square trinomial. $c = 9$

63. ERROR ANALYSIS Explain the error below. What is the correct product?
$(a - b)^2 = a^2 - b^2$ Possible answer: The square of a diff. is not the same as a diff. of squares; $a^2 - 2ab + b^2$.

(tr)Molly Eckler - puzzle, photo by Victoria Smith/HMH; (b)©Ruggero Vanni/CORBIS

PRACTICE A

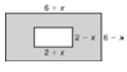
PRACTICE C

PRACTICE B

Multiply.

1. $(x + 2)^2$ $x^2 + 4x + 4$
2. $(m + 4)^2$ $m^2 + 8m + 16$
3. $(3 + a)^2$ $9 + 6a + a^2$
4. $(2x + 5)^2$ $4x^2 + 20x + 25$
5. $(3a + 2)^2$ $9a^2 + 12a + 4$
6. $(6 + 5b)^2$ $36 + 60b + 25b^2$
7. $(b - 3)^2$ $b^2 - 6b + 9$
8. $(8 - y)^2$ $64 - 16y + y^2$
9. $(a - 10)^2$ $a^2 - 20a + 100$
10. $(3x - 7)^2$ $9x^2 - 42x + 49$
11. $(4m - 9)^2$ $16m^2 - 72m + 81$
12. $(6 - 3n)^2$ $36 - 36n + 9n^2$
13. $(x + 3)(x - 3)$ $x^2 - 9$
14. $(8 + y)(8 - y)$ $64 - y^2$
15. $(x + 6)(x - 6)$ $x^2 - 36$
16. $(5x + 2)(5x - 2)$ $25x^2 - 4$
17. $(10x + 7y)(10x - 7y)$ $100x^2 - 49y^2$
18. $(x^2 + 3y)(x^2 - 3y)$ $x^4 - 9y^2$

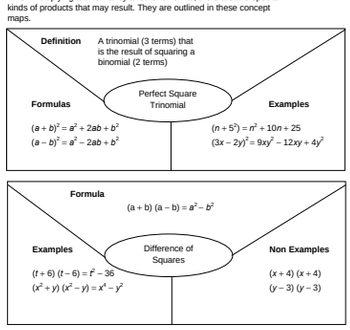
19. Write a simplified expression that represents the...
   a. area of the large rectangle. $36 - x^2$
   b. area of the small rectangle. $4 - x^2$
   c. area of the shaded area. 32



20. The small rectangle is made larger by adding 2 units to the length and 2 units to the width.
   a. What is the new area of the smaller rectangle? $16 - x^2$
   b. What is the area of the new shaded area? 20

READING STRATEGIES

When multiplying a binomial by another binomial, there are two special kinds of products that may result. They are outlined in these concept maps.



Answer the following.

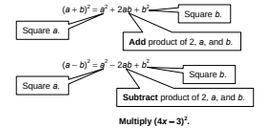
1. Which type of special product is $x^2 - 64y^2$? difference of squares
2. Which type of special product will result from multiplying $(x^2 - 3)(x^2 - 3)$? perfect square trinomial
3. Why is $(x + 4)(x + 4)$ not a difference of squares? It will have 3 terms.

Multiply. Then identify the type of special product.

4. $(c^2 + 10d)(c^2 + 10d)$ $c^4 + 20c^2d + 100d^2$ perfect square trinomial
5. $(2s + 3)(2s - 3)$ $4s^2 - 9$ difference of squares

RETEACH

A **perfect-square trinomial** is a trinomial that is the result of squaring a binomial.



Multiply $(x + 4)^2$.

$(x + 4)^2$	a: x b: 4
$x^2 + 2(x)(4) + 4^2$	*Middle term is added.*
$x^2 + 8x + 16$	*Simplify.*

Multiply $(4x - 3)^2$.

$(4x - 3)^2$	a: $4x$ b: 3
$16x^2 - 2(4x)(3) + 3^2$	*Middle term is subtracted.*
$16x^2 - 24x + 9$	*Simplify.*

State whether each product will result in a perfect-square trinomial.

1. $(x + 5)(x + 5)$ yes
2. $(x + 2)(x - 2)$ no
3. $(5x - 6)(5x - 6)$ yes

Fill in the blanks. Then write the perfect-square trinomial.

4. $(x + 7)^2$ Square a: x^2; $2(a)(b)$: 14x; Square b: 49; $x^2 + 14x + 49$
5. $(x - 1)^2$ Square a: x^2; $2(a)(b)$: 2x; Square b: 1; $x^2 - 2x + 1$
6. $(2x + 10)^2$ Square a: x^2; $2(a)(b)$: 40x; Square b: 100; $4x^2 + 40x + 100$

Multiply.

7. $(x - 8)^2$ $x^2 - 16x + 64$
8. $(x + 2)^2$ $x^2 + 4x + 4$
9. $(7x - 5)^2$ $49x^2 - 70x + 25$

MULTI-STEP TEST PREP

64. **a.** Michael is fencing part of his yard. He started with a square of length x on each side. He then added 3 feet to the length and subtracted 3 feet from the width. Make a sketch to show the fenced area with the length and width labeled.

b. Write a polynomial that represents the area of the fenced region. $x^2 - 9$

c. Michael bought a total of 48 feet of fencing. What is the area of his fenced region? 135 ft²

65. Critical Thinking The polynomial $ax^2 - 49$ is a difference of two squares. Find all possible values of a between 1 and 100 inclusive. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

66. Write About It When is the product of two binomials also a binomial? Explain and give an example. When 1 binomial is in the form $a + b$ and the other is in the form $a - b$; $(x + 2)(x - 2) = x^2 - 4$

TEST PREP

67. What is $(5x - 6y)(5x - 6y)$?

Ⓐ $25x^2 - 22xy + 36y^2$ Ⓒ $25x^2 + 22xy + 36y^2$

Ⓑ $25x^2 - 60xy + 36y^2$ Ⓓ $25x^2 + 60xy + 36y^2$

$25x^2$	$10x$
$10x$	4

68. Which product is represented by the model?

Ⓕ $(2x + 5)(2x + 5)$ Ⓗ $(5x + 2)(5x - 2)$

Ⓖ $(5x - 2)(5x - 2)$ Ⓙ $(5x + 2)(5x + 2)$

69. If $a + b = 12$ and $a^2 - b^2 = 96$ what is the value of a?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 10

70. If $rs = 15$ and $(r + s)^2 = 64$, what is the value of $r^2 + s^2$?

Ⓕ 25 Ⓖ 30 Ⓗ 34 Ⓙ 49

CHALLENGE AND EXTEND

71. Multiply $(x + 4)(x + 4)(x - 4)$.

72. Multiply $(x + 4)(x - 4)(x - 4)$.

71. $x^3 + 4x^2 - 16x - 64$

72. $x^3 - 4x^2 - 16x + 64$

73. If $x^2 + bx + c$ is a perfect-square trinomial, what is the relationship between b and c?

73. $b = \pm 2\sqrt{c}$

74. You can multiply two numbers by rewriting the numbers as the difference of two squares. For example:

$$36 \cdot 24 = (30 + 6)(30 - 6) = 30^2 - 6^2 = 900 - 36 = 864$$

Use this method to multiply 27 • 19. Explain how you rewrote the numbers. 513; rewrite 27 as 23 + 4 and 19 as 23 − 4.

HMH

MULTI-STEP TEST PREP **Exercise 64** involves finding area using polynomials. This exercise prepares students for the Multi-Step Test Prep.

TEST PREP DOCTOR When multiplying binomials of the form $(a - b)(a - b)$ in **Exercise 67,** the middle term of the product is negative. Choices **C** and **D** can be eliminated.

For **Exercise 69,** $a^2 - b^2 = 96$ is the difference of two squares, so $(a + b)(a - b) = 96$, and by substitution, $12(a - b) = 96$. After dividing both sides by 12, $a - b = 8$. This equation and $a + b = 12$ form a system of linear equations, easily solved by elimination.

Journal

Have students describe three ways to find the product $(x + 2)(x + 2)$; one method must use the special products rule.

ALTERNATIVE ASSESSMENT

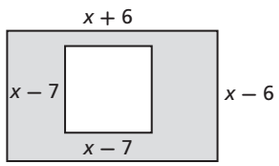
Have students create binomials of the form $(a + b)^2$, $(a - b)^2$, and $(a + b)(a - b)$; find each product; and describe how the special products rule is illustrated.

Power Presentations with PowerPoint®

6-6 Lesson Quiz

Multiply.

1. $(x + 7)^2$ $x^2 + 14x + 49$
2. $(x - 2)^2$ $x^2 - 4x + 4$
3. $(5x + 2y)^2$ $25x^2 + 20xy + 4y^2$
4. $(2x - 9y)^2$ $4x^2 - 36xy + 81y^2$
5. $(4x + 5y)(4x - 5y)$ $16x^2 - 25y^2$
6. $(m^2 + 2n)(m^2 - 2n)$ $m^4 - 4n^2$
7. Write a polynomial that represents the shaded area of the figure below. $14x - 85$



Also available online

PROBLEM SOLVING

Write the correct answer.

1. This week Kyara worked $x + 4$ hours. She is paid $x - 4$ dollars per hour. Write a polynomial for the amount that Kyara earned this week. Then calculate her pay if $x = 12$.
$x^2 - 16$
$128

2. A museum set aside part of a large gallery for a special exhibit.

Write a polynomial for the area of the gallery that is not part of the exhibit. Then calculate the area of that section if $x = 60$.
$0.75x^2 - x - 65$
2575 square feet

3. Gary is building a square table for a kitchen. In his initial sketch, each side measured x inches. After rearranging some furniture, he realized he would have to add one foot to the length and remove one foot from the width and have a rectangular table instead. Write a polynomial to represent the area of the rectangular table.
$x^2 - 144$ in²

A fountain is in the center of a square garden. The radius of the fountain is $x - 2$ feet. The length of the garden is $2x + 4$ feet. Use this information and the diagram for questions 4 – 7. Select the best answer.

4. Which polynomial represents the area of the fountain?
A $2\pi x - 4\pi$ C $\pi x^2 - 4\pi$
B $\pi x^2 - 4\pi x - 4\pi$ D $\pi x^2 - 4\pi x + 4\pi$

5. Which polynomial represents the area of the garden, including the fountain?
F $4x^2 + 8$ H $4x^2 + 16$
G $4x^2 + 16x + 16$ J $4x^2 + 8x + 16$

6. Which polynomial represents the area of the garden *outside* the fountain? (Use 3.14 for π.)
A $0.86x^2 + 28.56x + 3.44$
B $0.86x^2 + 3.44x + 28.56$
C $7.14x^2 + 28.56x + 3.44$
D $7.14x^2 + 3.44x + 28.56$

7. A 3-ft wide path is built around the garden. Which expression represents the area of the path?
F $12x + 33$ H $4x^2 + 28x + 29$
G $24x + 84$ J $4x^2 + 40x + 100$

CHALLENGE

The expansions of $(a + b)^2$, $(a - b)^2$, and $(a + b)(a - b)$ show patterns involving a and b. You can discover other patterns involving a and b when you combine these special products.

In Exercises 1–5, use the given values of a and b to complete the table below.

	a	b	$(a + b)^2$	$(a - b)^2$	$(a + b)^2 - (a - b)^2$	ab
1.	–3	5	4	64	–60	–15
2.	–2	–4	36	4	32	8
3.	3	6	81	9	72	18
4.	4	7	121	9	112	28
5.	5	–8	9	169	–160	–40

6. a. From the table, how are the values of $(a + b)^2 - (a - b)^2$ and ab related?
The difference $(a + b)^2 - (a - b)^2$ is four times the product ab.
b. Based on your work in Exercises 1–5 and your answer to part **a**, write a simplification of $(a + b)^2 - (a - b)^2$. $4ab$
c. Use algebra to show that your response to part **b** is true.
$(a^2 + 2ab + b^2) - (a^2 - 2ab + b^2) = 4ab$

7. Using a table like the one that you completed in Exercises 1–5 and the values of a and b used there, write a simplification of $(a + b)^2 + (a - b)^2$. $(a + b)^2 + (a - b)^2 = 2a^2 + 2b^2$

Write a conjecture for the simplification of each sum or difference. Try to do as much of the simplification as you can mentally.

8. $[(a^2 + b^2) + (a^2 - b^2)]^2$
$4a^4$

9. $[(a^2 + b^2) - (a^2 - b^2)]^2$
$4b^4$

Simplify each expression.

10. $[(a^2 + b^2) + (a^2 - b^2)]^2$
$[(a^2 + b^2) + (a^2 - b^2)]^2$
$= (a^2 + a^2 + b^2 - b^2)^2$
$= (2a^2)^2 = 2^2a^4 = 4a^4$

11. $[(a^2 + b^2) - (a^2 - b^2)]^2$
$[(a^2 + b^2) - (a^2 - b^2)]^2$
$= (a^2 - a^2 + b^2 + b^2)^2$
$= (2b^2)^2 = 2^2b^4 = 4b^4$

SECTION 6B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

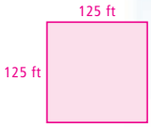
Polynomials

Don't Fence Me In James has 500 feet of fencing to enclose a rectangular region on his farm for some sheep.

1. Make a sketch of three possible regions that James could enclose and give the corresponding areas.
2. If the length of the region is x, find an expression for the width. $250 - x$
3. Use your answer to Problem 2 to write an equation for the area of the region. $A = x(250 - x)$
4. Graph your equation from Problem 3 on your calculator. Sketch the graph.
5. James wants his fenced region to have the largest area possible using 500 feet of fencing. Find this area using the graph or a table of values. $15{,}625\ \text{ft}^2$
6. What are the length and width of the region with the area from Problem 5? Describe this region. 125 ft × 125 ft; square

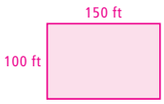
1. Possible answers:

$A = 15{,}625\ \text{ft}^2$

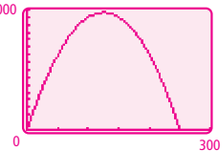


$A = 10{,}000\ \text{ft}^2$

$A = 15{,}000\ \text{ft}^2$



4.



(b), Ryan McVay/PhotoDisc/Getty Images; (tl), HMH; (tr), ©Chase Swift/Corbis

INTERVENTION

Scaffolding Questions

1. What is the perimeter of a rectangle? distance around the rectangle How is it calculated? $P = 2\ell + 2w$
2. What are some possible values for ℓ and w? Possible answers: $\ell = 40$, $w = 210$; $\ell = 30$, $w = 220$; $\ell = 45$, $w = 205$
3. What is the area of a rectangle? number of square units inside the rectangle How is it calculated? $A = \ell w$
4. What is an appropriate viewing window? Possible answer: $0 < x < 250$ and $0 < y < 16{,}000$
5. How can you find the greatest area from the graph or table? Graph: find the y-coordinate of the highest point; Table: find the greatest y-value.
6. How can you find the dimensions from the graph or table? Graph: find the x-coordinate of the highest point; Table: look for the greatest y-value and find its corresponding x-value.

Extension

Now James will use his fencing to create three sides of a rectangular region and build a wall for the fourth side. Now what is the greatest possible area? Describe this region. What is the minimum length of the wall? 31,250 ft²; 125 ft × 250 ft rectangle; 250 ft

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State Resources Online

READY TO GO ON?

CHAPTER 6

SECTION 6B

Quiz for Lessons 6-3 Through 6-6

1. $2r^6 + 4r^2 - 3r$; 2
2. $-8y^3 + y^2 + 2y + 7$; -8
3. $t^4 - 12t^3 - 4t$; 1

6-3 Polynomials

Write each polynomial in standard form and give the leading coefficient.

1. $4r^2 + 2r^6 - 3r$
2. $y^2 + 7 - 8y^3 + 2y$
3. $-12t^3 - 4t + t^4$
4. $n + 3 + 3n^2$ $3n^2 + n + 3$; 3
5. $2 + 3x^3$ $3x^3 + 2$; 3
6. $-3a^2 + 16 + a^7 + a$ $a^7 - 3a^2 + a + 16$; 1

Classify each polynomial according to its degree and number of terms.

7. $2x^3 + 5x - 4$
8. $5b^2$
9. $6p^2 + 3p - p^4 + 2p^3$
10. $x^2 + 12 - x$
11. $-2x^3 - 5 + x - 2x^7$
12. $5 - 6b^2 + b - 4b^4$

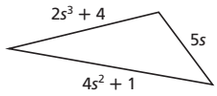
13. **Business** The function $C(x) = x^3 - 15x + 14$ gives the cost to manufacture x units of a product. What is the cost to manufacture 900 units? $728,986,514

6-4 Adding and Subtracting Polynomials

Add or subtract.

14. $(10m^3 + 4m^2) + (7m^2 + 3m)$
15. $(3t^2 - 2t) + (9t^2 + 4t - 6)$
16. $(12d^6 - 3d^2) + (2d^4 + 1)$
17. $(6y^3 + 4y^2) - (2y^2 + 3y)$
18. $(7n^2 - 3n) - (5n^2 + 5n)$
19. $(b^2 - 10) - (-5b^3 + 4b)$

20. **Geometry** The measures of the sides of a triangle are shown as polynomials. Write a simplified polynomial to represent the perimeter of the triangle. $2s^3 + 4s^2 + 5s + 5$



6-5 Multiplying Polynomials

Multiply.

21. $2h^3 \cdot 5h^5$
22. $(s^8t^4)(-6st^3)$
23. $2ab(5a^3 + 3a^2b)$
24. $(3k + 5)^2$
25. $(2x^3 + 3y)(4x^2 + y)$
26. $(p^2 + 3p)(9p^2 - 6p - 5)$

27. **Geometry** Write a simplified polynomial expression for the area of a parallelogram whose base is $(x + 7)$ units and whose height is $(x - 3)$ units. $(x^2 + 4x - 21)$ square units

6-6 Special Products of Binomials

Multiply.

28. $(d + 9)^2$
29. $(3 + 2t)^2$
30. $(2x + 5y)^2$
31. $(m - 4)^2$
32. $(a - b)^2$
33. $(3w - 1)^2$
34. $(c + 2)(c - 2)$
35. $(5r + 6)(5r - 6)$

36. **Sports** A child's basketball has a radius of $(x - 5)$ inches. Write a polynomial that represents the surface area of the basketball. (The formula for the surface area of a sphere is $S = 4\pi r^2$, where r represents the radius of the sphere.) Leave the symbol π in your answer. $(4\pi x^2 - 40\pi x + 100\pi)$ in^2

READY TO GO ON?

SECTION 6B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

7–12, 14–19. See Additional Answers.

21–26, 28–35. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 6B		
	Worksheets	Online	
Lesson 6-3	6-3 Intervention	Activity 6-3	Diagnose and Prescribe Online
Lesson 6-4	6-4 Intervention	Activity 6-4	
Lesson 6-5	6-5 Intervention	Activity 6-5	
Lesson 6-6	6-6 Intervention	Activity 6-6	

Ready to Go On? Enrichment, Section 6B
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Countdown

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

Answers

1. cubic
2. standard form of a polynomial
3. monomial
4. trinomial
5. $\frac{1}{32}$ in.
6. 1
7. 1
8. $\frac{1}{125}$
9. $\frac{1}{10{,}000}$, or 0.0001
10. $\frac{1}{16}$
11. $\frac{1}{256}$
12. $\frac{27}{4}$
13. $\frac{1}{m^2}$
14. b
15. $-\frac{1}{2x^2y^4}$
16. $2b^6c^4$
17. $\frac{3a^2}{4c^2}$
18. $\frac{s^3}{qr^2}$

Vocabulary

binomial	index	quadratic
cubic	leading coefficient	standard form of a polynomial
degree of a monomial	monomial	trinomial
degree of a polynomial	perfect-square trinomial	
difference of two squares	polynomial	

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ polynomial is a polynomial of degree 3.
2. When a polynomial is written with the terms in order from highest to lowest degree, it is in ___?___.
3. A(n) ___?___ is a number, a variable, or a product of numbers and variables with whole-number exponents.
4. A(n) ___?___ is a polynomial with three terms.

6-1 Integer Exponents

EXAMPLES

Simplify.

■ -2^{-4}

$-2^{-4} = -\frac{1}{2^4} = -\frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{1}{16}$

■ 3^0

$3^0 = 1$ *Any nonzero number raised to the zero power is 1.*

■ **Evaluate r^3s^{-4} for $r = -3$ and $s = 2$.**

r^3s^{-4}

$(-3)^3(2)^{-4} = \frac{(-3)(-3)(-3)}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{27}{16}$

■ **Simplify $\frac{a^{-3}b^4}{c^{-2}}$.**

$\frac{a^{-3}b^4}{c^{-2}} = \frac{b^4c^2}{a^3}$

EXERCISES

5. The diameter of a certain bearing is 2^{-5} in. Simplify this expression.

Simplify.

6. $(3.6)^0$
7. $(-1)^{-4}$
8. 5^{-3}
9. 10^{-4}

Evaluate each expression for the given value(s) of the variable(s).

10. b^{-4} for $b = 2$
11. $\left(\frac{2}{5}b\right)^{-4}$ for $b = 10$
12. $-2p^3q^{-3}$ for $p = 3$ and $q = -2$

Simplify.

13. m^{-2}
14. bc^0
15. $-\frac{1}{2}x^{-2}y^{-4}$
16. $\frac{2b^6}{c^{-4}}$
17. $\frac{3a^2c^{-2}}{4b^0}$
18. $\frac{q^{-1}r^{-2}}{s^{-3}}$

6-2 Rational Exponents

EXAMPLES

■ **Simplify $\sqrt[3]{r^6 s^{12}}$.**

$\sqrt[3]{r^6 s^{12}} = (r^6 s^{12})^{\frac{1}{3}}$ *Definition of $b^{\frac{1}{n}}$*

$= (r^6)^{\frac{1}{3}} \cdot (s^{12})^{\frac{1}{3}}$ *Power of a Product Property*

$= \left(r^{6 \cdot \frac{1}{3}}\right) \cdot \left(s^{12 \cdot \frac{1}{3}}\right)$ *Power of a Power Property*

$= (r^2) \cdot (s^4)$ *Simplify exponents.*

$= r^2 s^4$

EXERCISES

Simplify each expression.

19. $81^{\frac{1}{2}}$

20. $343^{\frac{1}{3}}$

21. $64^{\frac{2}{3}}$

22. $(2^6)^{\frac{1}{2}}$

Simplify each expression. All variables represent nonnegative numbers.

23. $\sqrt[5]{z^{10}}$

24. $\sqrt[3]{125x^6}$

25. $\sqrt{x^8 y^6}$

26. $\sqrt[3]{m^6 n^{12}}$

6-3 Polynomials

EXAMPLES

■ **Find the degree of the polynomial $3x^2 + 8x^5$.**

$3x^2 + 8x^5$ *$8x^5$ has the highest degree.*

The degree is 5.

■ **Classify the polynomial $y^3 - 2y$ according to its degree and number of terms.**

Degree: 3
Terms: 2

The polynomial $y^3 - 2y$ is a **cubic binomial**.

EXERCISES

Find the degree of each polynomial.

27. 5

28. $8st^3$

29. $3z^6$

30. $6h$

Write each polynomial in standard form. Then give the leading coefficient.

31. $2n - 4 + 3n^2$

32. $2a - a^4 - a^6 + 3a^3$

Classify each polynomial according to its degree and number of terms.

33. $2s - 6$

34. $-8p^5$

35. $-m^4 - m^2 - 1$

36. 2

Answers

19. 9
20. 7
21. 16
22. 8
23. z^2
24. $5x^2$
25. x^4y^3
26. m^2n^4
27. 0
28. 4
29. 6
30. 1
31. $3n^2 + 2n - 4$; 3
32. $-a^6 - a^4 + 3a^3 + 2a$; -1
33. linear binomial
34. quintic monomial
35. quartic trinomial
36. constant monomial

Study Guide: Review **443**

Answers

37. $-4t + 3$
38. $-6x^6 - x^5$
39. $3h^3 - 3h^2 + 5$
40. $2m^2 - 5m - 1$
41. $p^2 + 5p + 8$
42. $-7z^2 - z + 10$
43. $3g^2 + 2g + 4$
44. $-x^2 + 4x + 8$
45. $8r^2$
46. $6a^6b$
47. $18x^3y^2$
48. $3s^6t^{14}$
49. $2x^2 - 8x + 12$
50. $-3a^2b^2 + 6a^3b^2 - 15a^2b$
51. $a^2 - 3a - 18$
52. $b^2 - 6b - 27$
53. $x^2 - 12x + 20$
54. $t^2 - 1$
55. $8q^2 + 34q + 30$
56. $20g^2 - 37g + 8$

6-4 Adding and Subtracting Polynomials

EXAMPLES

Add.

■ $(h^3 - 2h) + (3h^2 + 4h) - 2h^3$
$(h^3 - 2h) + (3h^2 + 4h) - 2h^3$
$(h^3 - 2h^3) + (3h^2) + (4h - 2h)$
$-h^3 + 3h^2 + 2h$

Subtract.

■ $(n^3 + 5 - 6n^2) - (3n^2 - 7)$
$(n^3 + 5 - 6n^2) + (-3n^2 + 7)$
$(n^3 + 5 - 6n^2) + (-3n^2 + 7)$
$n^3 + (-6n^2 - 3n^2) + (5 + 7)$
$n^3 - 9n^2 + 12$

EXERCISES

Add or subtract.

37. $3t + 5 - 7t - 2$
38. $4x^5 - 6x^6 + 2x^5 - 7x^5$
39. $-h^3 - 2h^2 + 4h^3 - h^2 + 5$
40. $(3m - 7) + (2m^2 - 8m + 6)$
41. $(12 + 6p) - (p - p^2 + 4)$
42. $(3z - 9z^2 + 2) + (2z^2 - 4z + 8)$
43. $(10g - g^2 + 3) - (-4g^2 + 8g - 1)$
44. $(-5x^3 + 2x^2 - x + 5) - (-5x^3 + 3x^2 - 5x - 3)$

6-5 Multiplying Polynomials

EXAMPLES

Multiply.

■ $(2x - 4)(3x + 5)$
$2x(3x) + 2x(5) - 4(3x) - 4(5)$
$6x^2 + 10x - 12x - 20$
$6x^2 - 2x - 20$

■ $(b - 2)(b^2 + 4b - 5)$
$b(b^2) + b(4b) - b(5) - 2(b^2) - 2(4b) - 2(-5)$
$b^3 + 4b^2 - 5b - 2b^2 - 8b + 10$
$b^3 + 2b^2 - 13b + 10$

EXERCISES

Multiply.

45. $(2r)(4r)$
46. $(3a^5)(2ab)$
47. $(-3xy)(-6x^2y)$
48. $(3s^3t^2)(2st^4)\left(\frac{1}{2}s^2t^8\right)$
49. $2(x^2 - 4x + 6)$
50. $-3ab(ab - 2a^2b + 5a)$
51. $(a + 3)(a - 6)$
52. $(b - 9)(b + 3)$
53. $(x - 10)(x - 2)$
54. $(t - 1)(t + 1)$
55. $(2q + 6)(4q + 5)$
56. $(5g - 8)(4g - 1)$

6-6 Special Products of Binomials

EXAMPLES

Multiply.

- $(2h - 6)^2$

  $(2h - 6)^2 = (2h)^2 + 2(2h)(-6) + (-6)^2$

  $4h^2 - 24h + 36$

- $(4x - 3)(4x + 3)$

  $(4x - 3)(4x + 3) = (4x)^2 - 3^2$

  $16x^2 - 9$

EXERCISES

Multiply.

57. $(p - 4)^2$

58. $(x + 12)^2$

59. $(m + 6)^2$

60. $(3c + 7)^2$

61. $(2r - 1)^2$

62. $(3a - b)^2$

63. $(2n - 5)^2$

64. $(h - 13)^2$

65. $(x - 1)(x + 1)$

66. $(z + 15)(z - 15)$

67. $(c^2 - d)(c^2 + d)$

68. $(3k^2 + 7)(3k^2 - 7)$

Answers

57. $p^2 - 8p + 16$

58. $x^2 + 24x + 144$

59. $m^2 + 12m + 36$

60. $9c^2 + 42c + 49$

61. $4r^2 - 4r + 1$

62. $9a^2 - 6ab + b^2$

63. $4n^2 - 20n + 25$

64. $h^2 - 26h + 169$

65. $x^2 - 1$

66. $z^2 - 225$

67. $c^4 - d^2$

68. $9k^4 - 49$

Study Guide: Review **445**

CHAPTER 6

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

INTERACTIVE **Online Edition**

Resources

Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

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CHAPTER 6

CHAPTER TEST

Evaluate each expression for the given value(s) of the variable(s).

1. $\left(\frac{1}{3}b\right)^{-2}$ for $b = 12$ $\frac{1}{16}$
2. $(14 - a^0b^2)^{-3}$ for $a = -2$ and $b = 4$ $-\frac{1}{8}$

Simplify.

3. $2r^{-3}$ $\frac{2}{r^3}$
4. $-3f^0g^{-1}$ $-\frac{3}{g}$
5. m^2n^{-3} $\frac{m^2}{n^3}$
6. $\frac{1}{2}s^{-5}t^3$ $\frac{t^3}{2s^5}$
7. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of a cone when $\ell = 5$ cm and $r = 3$ cm. $75.36\ \text{cm}^2$

Simplify each expression. All variables represent nonnegative numbers.

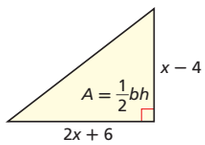
8. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$ $\frac{3}{5}$
9. $\sqrt[3]{43^3}$ 43
10. $\sqrt{25y^8}$ $5y^4$
11. $\sqrt[5]{3^5t^{10}}$ $3t^2$

Add or subtract.

12. $3a - 4b + 2a$ $5a - 4b$
13. $(2b^2 - 4b^3) - (6b^3 + 8b^2)$ $-10b^3 - 6b^2$
14. $-9g^2 + 3g - 4g^3 - 2g + 3g^2 - 4$ $-4g^3 - 6g^2 + g - 4$

Multiply.

15. $-5(r^2s - 6)$ $-5r^2s + 30$
16. $(2t - 7)(t + 4)$ $2t^2 + t - 28$
17. $(4g - 1)(4g^2 - 5g - 3)$ $16g^3 - 24g^2 - 7g + 3$
18. $(m + 6)^2$ $m^2 + 12m + 36$
19. $(3t - 7)(3t + 7)$ $9t^2 - 49$
20. $(3x^2 - 7)^2$ $9x^4 - 42x^2 + 49$
21. **Carpentry** Carpenters use a tool called a *speed square* to help them mark right angles. A speed square is a right triangle.



   a. Write a polynomial that represents the area of the speed square shown. $x^2 - x - 12$
   b. Find the area when $x = 4.5$ in. $3.75\ \text{in}^2$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 6

FOCUS ON SAT

When you receive your SAT scores, you will find a percentile for each score. The percentile tells you what percent of students scored lower than you on the same test. Your percentile at the national and state levels may differ because of the different groups being compared.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

HOT TIP!

You may use some types of calculators on the math section of the SAT. For about 40% of the test items, a graphing calculator is recommended. Bring a calculator that you are comfortable using. You won't have time to figure out how a new calculator works.

1. If $(x+1)(x+4)-(x-1)(x-2)=0$, what is the value of x?

(A) -1

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{4}$

(E) 1

2. Which of the following is equal to 4^5?

I. $3^5 \times 1^5$

II. 2^{10}

III. $4^0 \times 4^5$

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

3. If $x^{-4}=81$, then $x=$

(A) -3

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) 3

(E) 9

4. What is the value of $2x^3-4x^2+3x+1$ when $x=-2$?

(A) -37

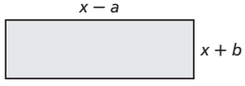
(B) -25

(C) -5

(D) 7

(E) 27

5. What is the area of a rectangle with a length of $x-a$ and a width of $x+b$?



(A) x^2-a^2

(B) x^2+b^2

(C) $x^2-abx+ab$

(D) $x^2-ax-bx-ab$

(E) $x^2+bx-ax-ab$

6. For integers greater than 0, define the following operations.

$a \square b = 2a^2+3b$

$a \triangle b = 5a^2-2b$

What is $(a \square b)+(a \triangle b)$?

(A) $7a^2+b$

(B) $-3a^2+5b$

(C) $7a^2-b$

(D) $3a^2-5b$

(E) $-3a^2-b$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 6

Organizer

Objective: Provide practice for college entrance exams such as the SAT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the SAT represent the following math content areas:

Number and Operations, 20–25%

Algebra and Functions, 35–40%

Geometry and Measurement, 25–30%

Data Analysis, Statistics, and Probability, 10–15%

Items on this page focus on:

- Algebra and Functions
- Geometry and Measurement

TEST PREP DOCTOR

1. Students who chose **A** or **E** may have made a sign error in the second term. Suggest that students use parentheses around each product of binomials until the multiplication is complete and then distribute the negative sign from the subtraction of the terms.

2. Students who chose **A, C,** or **E** may have added the bases in choice **I** and kept the exponent the same. Remind students that if the bases are the same when powers are multiplied, then they can add the exponents.

3. Students who chose **D** found the value of x if $x^4=81$. Suggest that students rewrite x^{-x} with a positive exponent and try again.

4. Students who chose **C** probably made a sign error in the first or second term. Remind students to be careful when using exponents with negative numbers. Be sure they understand when the negative sign is part of the base, and when it is not.

5. Students who did not choose **E** should review multiplying binomials. Remind students to be cautious of signs.

6. Students who chose **B** subtracted the two binomials instead of adding them. Remind students to read each test item carefully.

CHAPTER 6 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler explains how test item diagrams can inadvertently mislead students. If students assume that visual information from a diagram is correct, they will likely misinterpret the information. Advise students that diagrams are not always drawn to scale. Students should not rely solely on the appearance of the drawing to answer the test item. Instead they should look closely at a drawing's labels. They may even need to redraw the diagram to scale in order to better represent the problem. Explain that even though a test item may not include a diagram, it may be beneficial for students to make a quick sketch. Show students the importance of labeling their sketch with the information in the test item.

Any Question Type: Use a Diagram

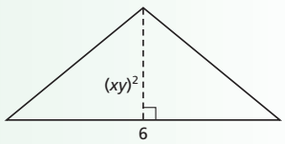
When a test item includes a diagram, use it to help solve the problem. Gather as much information from the drawing as possible. However, keep in mind that diagrams are not always drawn to scale and can be misleading.

EXAMPLE 1

Multiple Choice **What is the height of the triangle when $x = 4$ and $y = 1$?**

Ⓐ 2 Ⓒ 8

Ⓑ 4 Ⓓ 16



In the diagram, the height appears to be less than 6, so you might eliminate choices C and D. However, doing the math shows that the height is actually greater than 6. Do not rely solely on visual information. Always use the numbers given in the problem.

The height of the triangle is $(xy)^2$.

When $x = 4$ and $y = 1$, $(xy)^2 = (4 \cdot 1)^2 = (4)^2 = 16$.

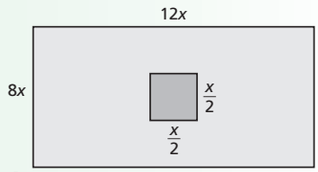
Choice D is the correct answer.

If a test item does not have a diagram, draw a quick sketch of the problem situation. Label your diagram with the data given in the problem.

EXAMPLE 2

Short Response **A square placemat is lying in the middle of a rectangular table. The side length of the placemat is $\left(\frac{x}{2}\right)$. The length of the table is $12x$, and the width is $8x$. Write a polynomial to represent the area of the placemat. Then write a polynomial to represent the area of the table that surrounds the placemat.**

Use the information in the problem to draw and label a diagram. Then write the polynomials.



area of placemat $= s^2 = \left(\frac{x}{2}\right)^2 = \left(\frac{x}{2}\right)\left(\frac{x}{2}\right) = \frac{x^2}{4}$

area of table $= \ell w = (12x)(8x) = 96x^2$

area of table $-$ area of placemat $= 96x^2 - \frac{x^2}{4} = \frac{384x^2 - x^2}{4} = \frac{383x^2}{4}$

The area of the placemat is $\frac{x^2}{4}$.

The area of the table that surrounds the placemat is $\frac{383x^2}{4}$.

HOT TIP! If a given diagram does not reflect the problem, draw a sketch that is more accurate. If a test item does not have a diagram, use the given information to sketch your own. Try to make your sketch as accurate as possible.

Read each test item and answer the questions that follow.

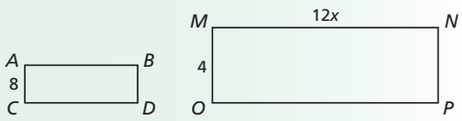
Item A

Short Response The width of a rectangle is 1.5 feet more than 4 times its length. Write a polynomial expression for the area of the rectangle. What is the area when the length is 16.75 feet?

1. What is the unknown measure in this problem?
2. How will drawing a diagram help you solve the problem?
3. Draw and label a sketch of the situation.

Item B

Multiple Choice Rectangle *ABDC* is similar to rectangle *MNPO*. If the width of rectangle *ABDC* is 8, what is its length?

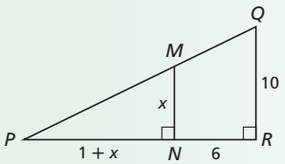


(A) 2
(B) $2x$
(C) $24x$
(D) 24

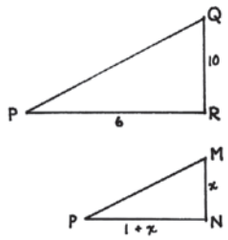
4. Look at the dimensions in the diagram. Do you think that the length of rectangle *ABDC* is greater or less than the length of rectangle *MNPO*?
5. Do you think the drawings reflect the information in the problem accurately? Why or why not?
6. Draw your own sketch to match the information in the problem.

Item C

Short Response Write a polynomial expression for the area of triangle *QRP*. Write a polynomial expression for the area of triangle *MNP*. Then use these expressions to write a polynomial expression for the area of *QRNM*.



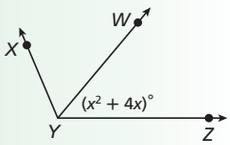
7. Describe how redrawing the figure can help you better understand the information in the problem.
8. After reading this test item, a student redrew the figure as shown below. Is this a correct interpretation of the original figure? Explain.



Item D

Multiple Choice The measure of angle *XYZ* is $(x^2 + 10x + 15)°$. What is the measure of angle *XYW*?

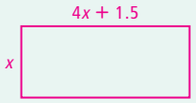
(F) $(6x + 15)°$
(G) $(2x^2 + 14x + 15)°$
(H) $(14x + 15)°$
(J) $(6x^2 + 15)°$

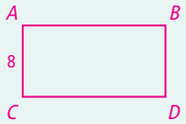


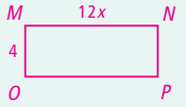
9. What information does the diagram provide that the problem does not?
10. Will the measure of angle *XYW* be less than or greater than the measure of angle *XYZ*? Explain.

Sam Dudgeon/HMH

Answers

1. the width
2. Possible answer: By drawing a rectangle and labeling each dimension, you will have a better understanding of what should be substituted into the area formula for length and width.
3. 

4. greater
5. No; it appears that the length of rectangle *ABDC* is less than the length of rectangle *MNPO*, and this is not consistent with the dimensions that are given.
6. Possible answer:




7. Possible answer: By redrawing the figures into 2 separate triangles, you can better see the base and height measures, which will help you correctly set up the area formula for each triangle.
8. No, it is not correct. The student mislabeled the base of triangle *QRP*. It should be labeled $7 + x$.
9. the measure of angle *WYZ*
10. Less than; angle *XYW* lies within angle *XYZ*, so angle *XYW* must be smaller.

Answers to Test Items

A. $4x^2 + 1.5x$; 1147.375 ft²
B. C
C. $5x + 35$; $\frac{1}{2}(x^2 + x)$; $-\frac{1}{2}x^2 + \frac{9}{2}x + 35$
D. F

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources
Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

Learn It Online
State Test Practice

CUMULATIVE ASSESSMENT

Multiple Choice

1. A negative number is raised to a power. The result is a negative number. What do you know about the power?

(A) It is an even number.
(B) It is an odd number.
(C) It is zero.
(D) It is a whole number.

2. Which expression represents the phrase *eight less than the product of a number and two*?

(F) $2 - 8x$
(G) $8 - 2x$
(H) $2x - 8$
(J) $\frac{x}{2} - 8$

3. An Internet service provider charges a $20 set-up fee plus $12 per month. A competitor charges $15 per month. Which equation can you use to find x, the number of months when the total charge will be the same for both companies?

(A) $15 = 20 + 12x$
(B) $20 + 12x = 15x$
(C) $20x + 12 = 15x$
(D) $20 = 15x + 12x$

4. Which is a solution of the inequality $7 - 3(x - 3) > 2(x + 3)$?

(F) 0
(G) 2
(H) 5
(J) 12

5. One dose of Ted's medication contains 0.625 milligram, or $\frac{5}{8}$ milligram, of a drug. Which expression is equivalent to 0.625?

(A) $5(4)^{-2}$
(B) $5(2)^{-4}$
(C) $5(-2)^{3}$
(D) $5(2)^{-3}$

6. Find the product $(b - 8)^2$.

(F) $b^2 + 64$
(G) $b^2 - 64$
(H) $b^2 - 16b - 64$
(J) $b^2 - 16b + 64$

7. Janet is ordering game cartridges from an online retailer. The retailer's prices, including shipping and handling, are given in the table below.

Game Cartridges	Total Cost ($)
1	54.95
2	104.95
3	154.95
4	204.95

Which equation best describes the relationship between the total cost c and the number of game cartridges g?

(A) $c = 54.95g$
(B) $c = 51g + 0.95$
(C) $c = 50g + 4.95$
(D) $c = 51.65g$

8. Which equation describes a line parallel to $y = 5 - 2x$?

(F) $y = -2x + 8$
(G) $y = 2x - 5$
(H) $y = 5 + \frac{1}{2}x$
(J) $y = 5 - \frac{1}{2}x$

9. A square has sides of length $x - 4$. A rectangle has a length of $x + 2$ and a width of $2x - 1$. What is the total combined area of the square and the rectangle?

(A) $10x - 14$
(B) $4x - 3$
(C) $3x^2 - 5x + 14$
(D) $3x^2 + 3x - 18$

Answers

1. B
2. H
3. B
4. F
5. D
6. J
7. C
8. F
9. C
10. J
11. C
12. F
13. $\frac{3}{4}$, or 0.75
14. 2
15. 16
16a. $28.00
b. $19.60

HOT TIP! **Test writers develop multiple-choice test options with distracters. Distracters are incorrect options that are based on common student errors. Be cautious! Even if the answer you calculated is one of the options, it may not be the correct answer. Always check your work carefully.**

10. Jennifer has a pocketful of change, all in nickels and quarters. There are 11 coins with a total value of \$1.15. Which system of equations can you use to find the number of each type of coin?

Ⓕ $\begin{cases} n + q = 11 \\ n + q = 1.15 \end{cases}$

Ⓖ $\begin{cases} n + q = 11 \\ 5n + 25q = 1.15 \end{cases}$

Ⓗ $\begin{cases} 5n + 25q = 11 \\ n + q = 1.15 \end{cases}$

Ⓙ $\begin{cases} n + q = 11 \\ 0.05n + 0.25q = 1.15 \end{cases}$

11. Which of the following is a true statement?

Ⓐ $\left[(a^m)^n\right]^p = a^{m+n+p}$

Ⓑ $\left[(a^m)^n\right]^p = a^{mn+p}$

Ⓒ $\left[(a^m)^n\right]^p = a^{mnp}$

Ⓓ $\left[(a^m)^n\right]^p = (a^{m+n})^p$

12. The drama club needs to raise at least \$1400 for a field trip. The club was given \$150 by the school administration. Club members are selling key chains for \$5 each. Which inequality represents the number of key chains k that the drama club needs to sell to go on its field trip?

Ⓕ $150 + 5k \geq 1400$

Ⓖ $5k - 150 \geq 1400$

Ⓗ $5k + 150 \leq 1400$

Ⓙ $150 \leq 5k + 1400$

Gridded Response

13. Evaluate the expression $3b^{-2}c^0$ for $b = 2$ and $c = -3$.

14. What is the slope of the line described by $-3y = -6x - 12$?

15. The volume of a plastic cylinder is 64 cubic centimeters. A glass cylinder has the same height and a radius that is half that of the plastic cylinder. What is the volume in cubic centimeters of the glass cylinder?

Short Response

16. A sweater that normally sells for \$35 was marked down 20% and placed on the sale rack. Later, the sweater was marked down an additional 30% and placed on the clearance rack.

a. Find the price of the sweater while on the sale rack. Show your work.

b. Find the price of the sweater while on the clearance rack. Show your work.

17. A set of positive integers (a, b, c) is called a *Pythagorean triple* if $a^2 + b^2 = c^2$.

a. Find a^2, b^2, and c^2 when $a = 2x$, $b = x^2 - 1$, and $c = x^2 + 1$. Show your work.

b. Is $(2x, x^2 - 1, x^2 + 1)$ a Pythagorean triple? Explain your reasoning.

18. Ron is making an ice sculpture. The block of ice is in the shape of a rectangular prism with a length of $(x + 2)$ inches, a width of $(x - 2)$ inches, and a height of $2x$ inches.

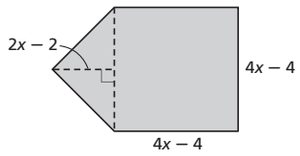
a. Write and simplify a polynomial expression for the volume of the block of ice. Show your work.

b. The final volume of the ice sculpture is $(x^3 + 4x^2 - 10x + 1)$ cubic inches. Write an expression for the volume of ice that Ron carved away. Show your work.

19. Simplify the expression $(3 \cdot a^2 \cdot b^{-4} \cdot a \cdot b^{-3})^{-3}$ using two different methods. Show that the results are the same.

Extended Response

20. Look at the pentagon below.



a. Write and simplify an expression that represents the area of the pentagon. Show your work or explain your answer.

b. Show one method of checking that your expression in part **a** is correct.

c. The triangular part of the pentagon can be rearranged to form a square. Write the area of this square as the square of a binomial.

d. Expand the product that you wrote in part **c**. What type of polynomial is this?

e. Is the square of a binomial ever a binomial? Explain your reasoning.

Short-Response Rubric

Items 16–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 20

4 Points = The student writes the correct expression with full work or explanation in part **a,** shows one method of checking the previous answer in part **b,** writes the correct area as a binomial square in part **c,** expands the product correctly and identifies the type of polynomial in part **d,** and answers correctly with explanation in part **e.**

3 Points = The student writes the correct expression with minimal work or explanation in part **a,** shows one method of checking the previous answer in part **b,** writes the area as a binomial square with minor errors in part **c,** expands the product with minor errors and identifies the type of polynomial in part **d,** and answers correctly with explanation in part **e.**

2 Points = The student answers parts **a, b,** and **e** correctly with attempted explanation; or the student answers parts **c, d,** and **e** correctly with attempted explanation.

1 Point = The student answers one part correctly but does not attempt all parts; or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

17a. $a^2 = (2x)^2 = 4x^2$
$b^2 = (x^2 - 1)^2 = x^4 - 2x^2 + 1$
$c^2 = (x^2 + 1)^2 = x^4 + 2x^2 + 1$

b. Yes; $(2x)^2 + (x^2 - 1)^2 = 4x^2 + x^4 - 2x^2 + 1 = x^4 + 2x^2 + 1$ and $(x^2 + 1)^2 = x^4 + 2x^2 + 1$. Because $(2x)^2 + (x^2 - 1)^2 = (x^2 + 1)^2$, the expressions form a Pythagorean triple.

18a. $(2x^3 - 8x)$ in^3

b. $(x^3 - 4x^2 + 2x - 1)$ in^3

19. $(3 \cdot a^2 \cdot b^{-4} \cdot a \cdot b^{-3})^{-3} = (3 \cdot a^3 \cdot b^{-7})^{-3} = 3^{-3} \cdot a^{-9} \cdot b^{21} = \frac{b^{21}}{27a^9}$;
$(3 \cdot a^2 \cdot b^{-4} \cdot a \cdot b^{-3})^{-3} = 3^{-3} \cdot a^{-6} \cdot b^{12} \cdot a^{-3} \cdot b^9 = 3^{-3} \cdot a^{-9} \cdot b^{21} = \frac{b^{21}}{27a^9}$

20a. $20x^2 - 40x + 20$

b. Possible answer: Substitute 5 for x in the diagram. Then the side lengths of the square are 16 units, and the height of the triangle is 8 units. The area of the square is 256 square units, and the area of the triangle is 64 square units. Add 256 to 64, which gives the area of the pentagon, 320 square units. Then substitute 5 for x in the simplified expression. It also simplifies to 320.

c. $(2x - 2)^2$

d. $4x^2 - 8x + 4$; trinomial

e. No; for any values of a and b, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. This is always a trinomial.